Two Illustrations of the Quantity Theory of Money Reloaded*

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Abstract

We review the relationship between inflation, nominal interest rates, and rates of money growth for a group of OECD countries. Once regime changes are isolated in the data, the behavior of these series maintains the close relationship predicted by standard quantity theory models. With an estimated model, we show those relationships to be relatively invariant to frictions that can deliver different short-run dynamics. The trend component obtained from statistical filters does reasonably well in capturing these regime changes in estimated models. The quantity theory relationships are alive and well, and thus they are useful for policy design aimed at controlling inflation.

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"These methods will yield clear results only if a good enough 'experiment' has been run by 'nature' over the sample period used."

"Two Illustrations of the Quantity Theory of Money" (p. 1013), Robert E. Lucas Jr, 1980.

1 Introduction

Are we on the verge of repeating the inflation of the '70s? Mainstream models in both academic and policy circles are mostly silent with respect to this question, since they almost exclusively focus on deviations from a steady state.[1]

This was a reasonable approach for the last few decades in most developed countries, since inflation indeed wandered around the targets established by central banks. The New Keynesian paradigm provided a theoretical framework for this policy approach, implicitly assuming that inflation had already been conquered. The focus of the analysis was how to use the cyclical properties of monetary policy to reduce the effects of potential distortions.[2]

However, in most developed economies, the recovery from the COVID-19 recession came with an increase in inflation rates, to levels that had not been seen since the early '80s. This created the fear that developed economies may be on the verge of another decade of stagflation like the one that started 50 years ago.

The purpose of this paper is to complement the NK apparatus with the quantity theory of money (QTM), which aims at explaining precisely what the New Keynesian apparatus slides under the rug: the determinants of long-run inflation.

Our object of study will be the high inflation in the '70s in several OECD economies. The reason to use those episodes is twofold. First, there appears to be a consensus that monetary forces are not relevant to understanding moderate inflationary episodes like those ones. For example, Sargent (1982) ends his paper with the following point:

"I have encountered the view that the events described here are so extreme and bizarre that they do not bear on the subject of inflation in the contemporary United States. On the contrary, it is precisely because the events were so extreme that they are relevant ... I believe that these incidents are full of lessons about our own, less drastic predicament with inflation, if only we interpret them correctly."

As we want to consider the chances of repeating the '70s, it is natural that we use them for our empirical analysis.

Second, in order to see the QTM prescriptions emerge in the data, we need "experiments" to be conducted by nature. By this, we mean, as Lucas (1980) does, events in which

[1] Notable exceptions will be discussed below.

[2] This agenda is described in detail in Woodford (2003).

inflation does exhibit sufficiently persistent fluctuations. It is only with large and persistent movements in inflation that there is a chance to detect co-movements between inflation and monetary policy variables.

In this paper, we search for those experiments during the high inflation episodes of the '70s and '80s in several OECD countries. We use the word "experiments" as Lucas does. Specifically, we do not mean circumstances in which only monetary forces are moving so one can isolate the causal effect of monetary policy. We mean events in which there are regime changes that drive movements in inflation over a medium term so we can see the extent to which over that period, inflation correlates with monetary policy variables.

The QTM has been interpreted in different ways. For Friedman, it meant a stable relationship between an appropriate measure of the total quantity of money, a short-term nominal interest rate and total nominal spending: a stable real money demand. Probably influenced by the experience of interest rate pegging and subsequent inflation that followed World War II, Friedman viewed a policy that controls the short-term interest rate as unstable, and favored a control of monetary aggregates. Under this view, the Fisher equation only determines the short-term nominal interest rate as a residual, and it plays no role in the determination of inflation. Thus, while his conceptual framework clearly included a Fisher equation, it played a secondary role.

Friedman's view has been criticized by many. In particular, the notion that the most convenient instrument for policy is the quantity of money has been seriously contested. In contrast to Friedman, the New Keynesian approach argues that the best way to control inflation is using the short-term interest rate. The Fisher equation plays a prominent role in inflation determination in those models, and it is the demand for real money that is a nuisance, useful only for the determination of the level of monetary aggregates in equilibrium. Therefore, it is argued that the money demand can be dispensed with in solving the model.

In his classic 1980 paper, Lucas proposed an agnostic view regarding which is the best instrument for policy and which instrument the Federal Reserve actually chose to control in the data he analyzed. Both the demand for real money and the Fisher equation are equilibrium conditions of a class of monetary models, and can then be jointly tested in the data. In doing so, one does not need to take a stand on which is the instrument chosen by policy. We follow Lucas's approach in viewing the QTM as the implication that both the demand for real money *and* the Fisher equation ought to hold in equilibrium. Considered this way, the QTM, whose aim is to understand the long-run behavior of inflation, is fully consistent with the NK program, whose aim is to understand the short-run behavior.

As mentioned above, to evaluate the theory, we use the experience of several OECD countries starting at the end of the '50s. By then, the inflation rate was very low, but by the mid '60s, it started to increase, reaching its highest value by the late '70s or early '80s, depending on the country. It then went down, and by the early 2000s, it reached the low levels that prevailed in the early '60s.

Figure 1 summarizes the rising inflation and its subsequent conquest. It depicts the average inflation rate for several OECD countries from 1960 to 2005, together with a one-standard-deviation band.

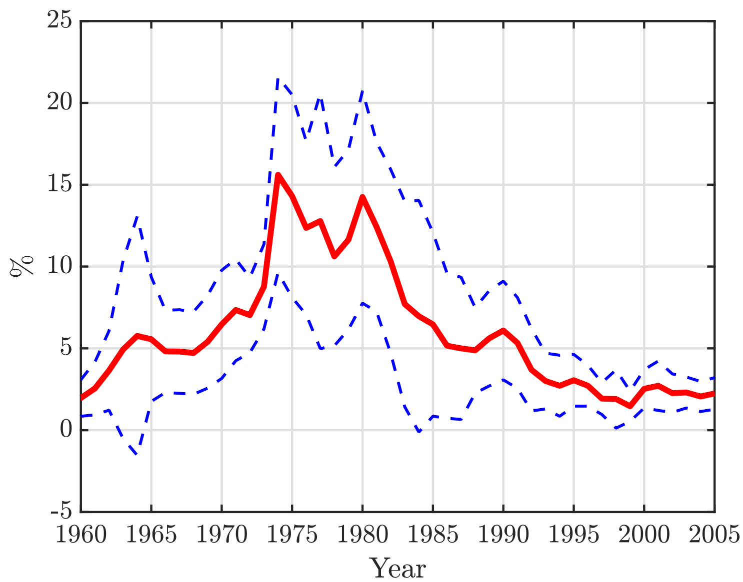


Figure 1: Average Inflation for 13 OECD Countries (1960–2005)
Notes: The 13 OECD countries include the United States, Australia, Canada, Germany, Denmark, Italy, Japan, the Netherlands, New Zealand, Portugal, South Korea, Spain, and the UK.

The analysis of this paper is consistent with the view that the two main laws of the QTM can explain the most relevant movements in the inflationary experiences depicted in Figure 1, while the details of the NK agenda matter little.

As a first approximation, we follow Lucas (1980) and take the two main laws of the quantity theory to mean that i) there is a one-to-one relationship between the nominal interest rates and the inflation rate, and ii) there is a one-to-one relationship between the growth rate of money and the inflation rate—once changes in the nominal interest rate are accounted for. To the extent that monetary policy can control the evolution of either the quantity of money or the short-term interest rate, the two main laws imply that the most relevant movements in inflation are explained by monetary forces.

As emphasized above and in the opening quote, events in which the underlying monetary policy regime changes over time can help identify the quantity theory laws in the data. To put it differently, unless the target for inflation set by the monetary authorities, either explicitly or implicitly, changes sufficiently over time, it will be difficult to detect the quantity theory implications. The reason is that beyond the monetary policy regime put in place, there are various shocks that affect the price level in an economy, among other variables. The properties of these shocks, together with the functioning of markets, determine the distribution of inflation rates in equilibrium. The main tenet of the quantity theory is that the mean of that distribution can be uniquely pinned down by monetary policy. To the extent that regime changes can bring about variation in the mean of that distribution, they make the data potentially informative about the quantity theory.[3]

This logic explains several exercises that have been performed in the literature. The first is Sargent's (1982) study of four hyperinflations in Europe following World War I, which were abruptly ended through changes in policy regimes. As with all successful attempts to end hyperinflations, there is ample independent evidence regarding the time and nature of the policy regime change. Sargent went one step further and explained the changes in the monetary policy regimes as the result of changes in the fiscal policy regimes. In this way, in all the cases he studies, he is able to establish the details of the fiscal and monetary reforms as well as the timing.

A second exercise is the cross-country analysis, which averages data over long periods of time and includes in the sample countries with different monetary policy regimes. It can be found in Vogel (1974), McCandless and Weber (1995) and, more recently, Teles, Uhlig, and Valle e Azevedo (2016).

The third one is the strategy proposed in Lucas (1980), from which our opening quote is taken. We heavily borrow from Lucas (1980), as attested to by the title of this paper. His strategy is to use a filter, with the hope that by removing from the data the effect of short-lived policy reactions to short-lived shocks, the effect of the more persistent regime change will dominate the fluctuations in inflation. Lucas defends this strategy as follows:

"One could in principle test the neoclassical laws by deriving their implications for the parameters of a structural econometric model. This course, while attractive in theory ... it involves nesting the two hypotheses in question within a complex maintained hypothesis, which must be accepted as valid in order to carry out the test. The virtue of relatively atheoretical tests ... is that they correspond to our theoretically based intuition that the quantity theoretic laws are consistent with a wide variety of possible structures."

The evidence presented in Figure 1 suggests monetary policy regimes that evolved over time and is therefore fertile land for our exploration.

[3]This point echoes a related point made by King and Watson (1997) that a process for money that is stationary—that is, a process with no long-run variability—cannot generate useful data to test the neutrality of money.

We go beyond the analysis in Lucas (1980) in several ways. First, we take a stand on specific way to separate the data between "trend" and "cycle", which gives us a precise definition of what we mean by "the most relevant" movements in inflation. Specifically, we use a narrative that isolates tightening cycles in the federal funds rate for the United States since 1960. We identify those cycles using that narrative, and calibrate the filter so as to remove just those tightening cycles.

Second, we move in the direction that Lucas disregards in the previous quote: we test the quantity theory implications in a particular structural model. Specifically, we conduct a structural estimation of a New Keynesian monetary model on United States data, in which we allow, but do not impose, monetary policy regime changes. A regime change in monetary policy can be estimated quite precisely.

Using the estimated model and estimated magnitude of the regime changes, we show in simulated data that the filter, calibrated as discussed above, captures the estimated regime change very well. We also show that frictions in the setting of prices, typically present in New Keynesian models, play a minimal role in explaining fluctuations in inflation. We then argue that the regime change becomes visible after monetary policy cycles are removed from the data. In the United States, monetary policy cycles typically last between two and six years, with an average duration of roughly four years. Once those monetary policy cycles are filtered away, the quantity theory relationships emerge.

But this, of course, need not be the case—if there were no regime change, removing monetary policy cycles would not reveal the quantity theory relationships. This is confirmed in our simulations, by applying our filter to the simulated model, when the regime change is shut down.

Finally, we extend the sample by adding four more decades of data and by offering an international perspective using data from several other countries that to some extent are comparable to the United Sates. We apply the same filter, calibrated according to the narrative described above and tested in the simulated model, to all the data for all countries. The filtered data for most of the countries show a notable resemblance with the QTM implications. Interestingly, the notable exceptions are Germany and the Netherlands, the two countries for which inflation did not go up persistently during the '70s and '80s, suggesting the lack of a regime change in those countries.

Our results challenge prevailing narratives of the high inflation episode in the '70s and '80s in the United States, as well as in some other countries we analyze. These accounts base their explanations on the role of the Phillips curve in shaping inflation but do so with models in which the inflation target remains fixed for the whole sample. An example is the pioneering analysis in Galí and Gertler (1999), which opened up a large and influential empirical literature—see, for instance, Smets and Wouters (2007). According to this narrative, the great inflation episodes in developed economies are the result of real shocks interacting with frictions in the setting of prices, rather than the monetary forces of the quantity theory. And the substantial losses of output observed during the early '80s were key drivers of the fall in inflation.

By contrast, the results of our paper imply that inflation fell as a consequence of a regime switch in monetary policy, while price frictions played no substantive role. In this sense, our message is similar to the ones in Ireland (2007) and Uribe (2022), who allow for permanent shocks to the inflation target and show that the low-frequency changes in inflation are the consequence of permanent shocks. We discuss how our approach complements their work below. Our interpretation is also consistent with the main argument in Hazell, Herreño, Nakamura, and Steinsson (2022), in which time variation in long-run inflation expectations drives the most relevant movements in inflation. As we show in Appendix C, the regime change in monetary policy we identify in estimating the model—which drives inflation expectations in the model—very closely follows their measure of long-run inflation expectations—which drives medium-term inflation in their model.

A final caveat is in order. Our analysis makes no attempt at explaining why there were regime changes in monetary policy. Our estimated regime changes are consistent with the explanation in Sargent (2001), based on a monetary authority that learns over time the slope of the Phillips curve, using statistical methods. But it is also consistent with an alternative explanation provided more recently by Bianchi and Melosi (2022), who argue that the rise in inflation is the result of fiscal expansions during a fiscally led regime. The shock that drives the high inflation of the late '70s, by their interpretation, is consistent with the regime change we estimate.[4]

Related Literature Our paper is closely related to Ireland (2007), who estimates a cashless model with no regime change, but allows for permanent shocks to the inflation target. It is also in line with the analysis of Uribe (2022), which makes a critical distinction between permanent and transitory shocks to monetary policy. There are three main differences between our analysis and theirs. First, we consider monetary models with money, so we jointly study the relationship between nominal interest rates and inflation and the relationship between the growth of money stock and inflation. Second, rather than relying on permanent shocks, as both Ireland (2007) and Uribe (2022) do, we estimate a regime change in monetary policy and let the data inform us on both the parameters

[4] In their estimated model, cost push shocks also play a minor role in explaining the longer-term movements in inflation, relative to the previous literature.

governing the change as well as its timing. We prefer this approach, since otherwise one needs to rely on a sequence of positive shocks to explain the rise in inflation, then a sequence of negative shocks to explain the reduction in inflation and then a sequence of shocks equal to zero, to explain a constant target in the last two decades and a half of our sample. This distinction, not very relevant in some dimensions, is important for some of the simulations, as we discuss in detail in Section 4.3. Finally, besides estimating the model using the US data, we study the quantity theory laws using filtered data for several OECD countries and use the estimated model to evaluate the choice of the filter used. Our results provide strong support to the notion that the chosen filter captures the regime change very well.

Our paper is also related to Reynard (2007), who studies the relationship between monetary aggregates and inflation in recent US monetary history. His focus is on the predictive power of policy stance measures using either interest rates or monetary aggregates, and he argues the latter perform better. His interest is on short-run movements, while we focus on more persistent changes. Our analysis of cases with very low nominal rates casts doubts on how applicable his methodology is for those cases.

Several papers have allowed for time-varying inflation trends in different contexts. For instance, Cogley and Sbordone (2008) show that once a drift is allowed in trend inflation, an otherwise standard DSGE model can account for the persistency in inflation without imposing ad-hoc indexation schemes. Justiniano, Primiceri, and Tambalotti (2013) also find that the persistent inflation target process is crucial to explaining the low-frequency inflation dynamics in the post-war period. Bianchi (2013) models the Markov-switching regime changes in both the monetary policy and stochastic volatility parameters and highlights the role of agents' beliefs. Del Negro, Giannone, Giannoni, and Tambalotti (2017) allow for stationary but very persistent shocks to trend inflation in order to estimate the natural real interest rate.

The notion that formally taking into account changes in trend inflation is important for understanding the term structure of interest rates has long been recognized in the finance literature. Hördahl, Tristani, and Vestin (2006) and Rudebusch and Wu (2008) explicitly investigate the effect that changes in trend inflation have on the yield curve. More recently, Bauer and Rudebusch (2020) show that allowing for time-varying trends in both inflation and the real interest rate is crucial for understanding the dynamics of treasury yields. As in Ireland (2007) and Uribe (2022), they allow for permanent shocks to the trend.

All these papers model shocks to the inflation trend as permanent shocks that can potentially happen in any period. And in all of them, not surprisingly, trend inflation starts growing in the early '70s and starts declining in the early '80s. Thus, what are modeled as permanent shocks that are i.i.d. happen to all be very close to zero in the '60s, all positive in the '70s, all negative in the '90s and then again all very close to zero ever since. Thus, rather than allowing for permanent shocks whose distribution is independent of time, we allow for regime changes and let the data inform us about their size and timing. And those regime changes are the ones that resemble the laboratory experiments mentioned in the opening quotes and are likely to be captured by the statistical filter we uniformly apply to a variety of countries.

A series of papers, including Sims and Zha (2006) and Liu, Waggoner, and Zha (2011), use Markov-switching (MS) regimes to model changes in trend inflation. We do not follow that route, for both theoretical and empirical reasons. On the theory side, we are interested in studying episodes with an apparent single regime change, like the high inflation of the '70s and '80s. Simulations from our estimated model are therefore well suited to appraise the ability of the statistical filter to capture such a regime change. Contrary to our strategy, and in a fashion similar to assuming permanent shocks, adopting a Markov-switching model would imply that there is a constant probability of repeating what our model identifies as the policy mistakes of the '70s. More importantly, from the empirical side, once we focus on a single episode, we find that a large fraction of the movements in inflation in the United States can be explained with purely monetary forces. In contrast, Liu, Waggoner, and Zha (2011) do not find a prominent role for those forces.

Our results highlight the more general point that long-run or steady state relations are difficult to detect in the absence of long-run or steady state variability. Barsky (1987) makes this point, focusing on the Fisherian correlation, and shows how the correlation depends on the underlying persistence of inflation; Sargent (1971) makes the point in the case of the long-run Phillips curve; King and Watson (1997) show that long-run neutrality can be assessed only if money is an integrated process, because otherwise there are no permanent changes in money, making data uninformative about the long-run neutrality of money; and, more recently, Sargent and Surico (2011) show that quantity theory relations may be detected when the monetary policy response to inflation is weak. In our approach, monetary regime changes stem from permanent shifts of the inflation target—a shift of the zero frequency—but the monetary policy rule coefficients remain the same (in our baseline). In our case, the most persistent movements in inflation come from regime changes in the inflation target, while in Sargent and Surico (2011), persistent movements in inflation arise because of a weak response of the monetary authority to inflation deviations from a stable inflation target. So although persistent movements could be generated differently, we share the general conclusion of McCallum (1984), Barsky (1987) and Sargent and Surico (2011) that persistent movements are required to detect the long-run relations of the quantity theory.

The paper proceeds as follows. In Section 2, we briefly discuss a very simple multi-country model and derive the two illustrations that guide our empirical analysis. In Section 3, we show how to pick the parameter in the Hodrick-Prescott filter that removes the tightening cycles of monetary policy in the United States. In Section 4, we estimate a New Keynesian model that is standard, except that we allow for regime changes in the inflation target and explicitly model and estimate a real money demand. In Section 5, we filter data for several OECD countries so as to remove short-term fluctuations and evaluate the ability of the two illustrations to match the data. A concluding section provides a broad discussion of the policy implications of the evidence discussed in the paper.

2 The Two Illustrations

We begin by presenting the two illustrations discussed in Lucas (1980) in a monetary open economy model. The model is very simple, and it is used to fix ideas, so we will focus only on some of the equilibrium conditions. The open economy feature is important, since we will use the interest parity condition in our empirical application discussed in Section 5.

The economy consists of multiple countries $j \in \{1, ..., J\}$. Only country 1 can issue international bonds to other countries.

The period utility for country j follows

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[u(c_{jt}^N) + u(c_{jt}^T)\right],$$

where c_{jt}^N is consumption of a home good that is not traded in international markets and c_{jt}^T is consumption of a traded good. The function u satisfies standard Inada conditions.

Money demand is induced by a cash-in-advance constraint,

$$M_{jt} n_{jt} \geq P_{jt}^N c_{jt}^N + P_{jt}^T c_{jt}^T,$$

where n_t is the number of trips to the bank that the household makes every period. Making trips to the bank entails a fixed stochastic cost γ_t in units of time. The total time endowment is equal to 1, so feasibility requires

$$\gamma_{jt} n_{jt} + h_{jt} + l_{jt} = 1,$$

where h_{jt} is the amount of labor devoted to the production of the non-traded good and l_{jt} is the amount of labor devoted to the production of the traded good.

Labor is the only input in production, and technologies in country j are such that

$$
\begin{aligned}
y_{jt}^{N} &= z_{jt}^{N} h_{jt}, \\
y_{jt}^{T} &= z_{jt}^{T} l_{jt},
\end{aligned}
$$

where z_{jt}^{N} and z_{jt}^{T} are stochastic productivity processes. The rest of the model details and the definition of the competitive equilibrium are available in Appendix A.

Derivation of the two illustrations We start by presenting the two illustrations for country 1, so we temporarily drop the j-indexes. The first illustration involves the relationship between real money balances as a proportion of output and the nominal interest rate in bonds, which takes the celebrated squared root formula derived by Baumol (1952) and Tobin (1956):

$$\frac{m_t}{c_t} = \sqrt{\frac{\gamma_t}{i_t}}. \tag{1}$$

In Appendix A, we show that this relationship holds as long as the cash-in-advance constraint is binding, which happens when the nominal interest rate is strictly positive. Assuming that the cash-in-advance constraint is binding is quite reasonable for the countries in the period considered in Figure 1, with the possible exception of Japan, which since 1995 has had near-zero interest rates. We discuss the case of Japan post-1990 in a separate subsection in which we also study the period of very low interest rates in a few other countries following the financial crisis of 2008-09.

We can further take the log difference of equation (1) to obtain a relationship between inflation π, the growth rate of the nominal quantity of money μ, the growth rates of output and the nominal interest rates, respectively denoted by g^x and g^i, plus the growth rate of the shock:

$$\pi_{t+1} = \mu_{t+1} - g_{t+1}^{x} + \frac{1}{2} g_{t+1}^{i} - \frac{1}{2} g_{t+1}^{\nu}. \tag{2}$$

The second illustration, also derived in detail in Appendix A, relates the nominal interest rate with the real interest rate and the expected inflation rate:

$$\mathbb{E}\left[(1 + r_{t+1})(1 + \pi_{t+1})\right] = 1 + i_t, \tag{3}$$

where r_{t+1} is a measure of the real interest rate of assets carried from period t to $t+1$.[5]

Using a log-linear approximation of equation (3) and replacing the inflation expectation

[5]This real interest rate is measured in terms of marginal utilities of real wealth, using the indirect utility function. In Appendix A, we show how this relates to a real interest rate measured in units of consumption, rather than in wealth.

with the inflation rate π_{t+1} and an error term ξ^{π}_{t+1}, we rewrite equation (3) as

$$i_t = \pi_{t+1} + r_{t+1} - \xi^{\pi}_{t+1}, \tag{4}$$

where ξ^{π}_{t+1} is a zero-mean shock, independent of any of the variables in the information set at time t, since it is an expectational error.

These two equilibrium relationships (1) and (3), or equivalently the log-linearized equations (2) and (4), involve three endogenous nominal variables, the inflation rate, the rate of money growth relative to output, and the nominal interest rate plus two endogenous real variables, the real interest rate and real output. With flexible prices, as we have assumed so far, the model is essentially neutral, in the sense that real variables are almost independent of nominal variables.[6]

Thus, we can interpret (1) and (3), or equivalently the log-linearized equations (2) and (4), as a system of two equations in three unknown variables, the inflation rate, the rate of money growth relative to output, and the nominal interest rate, given the behavior of the real variables, which are determined by the other equations of the model, as described in Appendix A.

These two equations do not fully characterize the behavior of prices, money and interest rates. Conspicuous by its absence is a description of monetary policy. This was a conscious choice, since according to the theory, the two implications ought to hold independently of whether the central bank adopts a money rule or an interest rate rule. Thus, to validate the empirical performance of the two illustrations in the next section, we do not need to take a stand on how monetary policy is executed.

For other countries $j \neq 1$, we obtain the same two equations (2) and (4) as for country 1. We also obtain a parity condition

$$\mathbb{E}[(1 + \pi_{jt+1})\frac{\xi_{jt+1}}{\xi_{jt}}(1 + r^{*}_{t+1})] = (1 + i_{jt}), \tag{5}$$

where $\xi_{jt} = \frac{S_{jt}P^{*}_{t}}{P_{jt}}$ is the real exchange rate. Equation (5) connects the real rate in country 1 to inflation and interest rates in country j.

[6]This is not a theoretical result, since inflation distorts the allocation of time in this economy. However, as shown in Cooley and Hansen (1989) for the values of inflation experienced in the United States changes in nominal variables barely affect the behavior of real variables.

3 Empirical Implementation: The Filter

Our theoretical assumptions (Baumol and Tobin's assumptions, really) pin down the coefficients on the right-hand side of (2). We use those values for all countries in our exercise in Section 5.[7] In studying particular countries, it should clearly be possible to do better, since there may be country-specific features implying different values for the elasticities. To a large extent, our conclusion will be that improvements in the fit of the theory, while worth making on a country-by-country basis, will bring modest progress to our ability to understand the most relevant movements in inflation for this group of countries as a whole.

Our model abstracts from all sorts of plausible frictions, so it has no hope of matching data in the short run. Therefore, in Section 5, we follow Lucas (1980), abandon that specific quest at the outset, and use a statistical filter capable of detecting long-run variability.[8]

There is a key degree of freedom that needs to be settled in following this strategy: the ability to split the data between the cyclical and the trend components. Lucas (1980) does not take a stand on that question. He chooses a family of filters that depend on a single parameter. The higher the value of that parameter, the lower is the frequency that is extracted from the data.

Lucas's paper is like a mystery movie. If you stare at the data, chaos prevails. But as the viewer moves along the sequence of plots, each retaining lower frequencies, the patterns start to emerge. By the time the reader arrives at the last plot, the two illustrations shine and order prevails over chaos. Just like the book of Genesis.

Our paper offers just a picture: we take a stand on a particular way to split the data. This, in turn, provides a specific definition of what we mean by "medium run." This definition clarifies for which policy questions our framework will not be useful and for which questions it may be.

To decompose the data, we use the Hodrick-Prescott (HP) filter. The filter's decomposition between the cyclical and the trend components is controlled by a single parameter, denoted by λ. By taking a stand on the value for λ, we take a stand on a particular way to decompose the data.

In the next section, we estimate a structural model subject to monetary policy regime changes that can shift the unconditional mean of nominal variables. Considered in this

[7] Equation (2) departs from Lucas (1980) and Benati (2009) in that they set the value of the interest rate elasticity to zero, rather than to one-half, as the Baumol-Tobin model implies. Had we followed their strategy, the fit of the model to the data would have worsened for most of the countries we analyze in Section 5.

[8] In any event, we present below both the "trend" component and the original data. Our eyes see the original data in a different way after observing the "trend" component—and yours also will, we hope.

way, our series are non-stationary, so the usual frequency domain decomposition is not valid. Each regime is covariance stationary, and so oscillations of all frequencies are present in each regime. Although we label the extracted components from the HP filter as "cycle" and "trend"—terms that are commonplace—it will become evident from the analysis of the structural model that regime changes that shift the unconditional mean get picked up by the trend component of the HP filter.[9] But it is important to keep in mind that the extracted "trend" does not belong to any one regime, so whatever the dominant frequency of this "trend" component happens to be, it is meaningless and should not be interpreted literally in the presence of regime changes. It is for this reason that in the main text, we prefer to use the HP filter, rather than a band pass filter, to avoid any potential confusion arising between the extracted frequency for the "trend" and the length of monetary policy cycles. An analysis with the band pass filter is provided in Appendix B.3.[10]

In order to discipline the choice of λ, we use the recent history of monetary policy in the USA. Specifically, we base our choice of λ on a particular narrative regarding the behavior of the short-term interest rate in the USA. We believe it to be a widely accepted narrative among macroeconomists. Figure 2(a) depicts the raw data series of the federal funds rate, which is further decomposed into a trend and a cyclical component using $\lambda = 6.5$ in Figure 2(b) and using $\lambda = 100$ in Figure 2(c). The relative merits of the two values for λ are discussed in detail below.

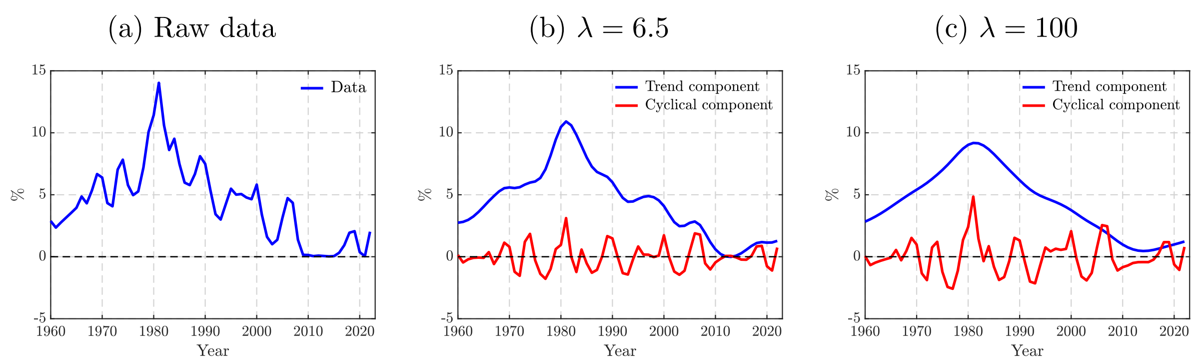


Figure 2: Nominal Interest Rates in the US

The key historical element to build the narrative is the notion of a "tightening cycle." Any such cycle is defined as a series of consecutive periods exhibiting increasing values for interest rates. These are clearly visible in Figure 2, more obviously so in the extracted cyclical component shown in panels (b) and (c). Particularly famous tightening cycles

[9] See Kulish and Pagan (2021) for a discussion on the distinction between cycles and oscillations.

[10] A band pass filter that preserves only data at frequency lower than 24 years removes the "tightening cycles" and has a similar performance as the HP-filter with a smoothing parameter of 100.

are the ones known as the "Volcker stabilization"—starting at the end of the '70s—and "Greenspan's conundrum"—the one that starts in 2004.[11]

The narrative interprets these cycles in the interest rate as a temporary policy response to temporary shocks, so as to stabilize the economy around certain desired values. This role of policy finds its strongest intellectual rationale in the New Keynesian literature, which emphasizes frictions in the setting of prices. In these models, price frictions generate only temporary effects on the equilibrium, which vanish "in the long run."

Although we do not need to take a stand on how policy is executed, it is convenient to consider an interest rate policy that, as in Taylor (1993), follows

$$i_t = i^* + \phi_\pi(\pi_t - \pi^*) + \phi_y(y_t - y^*) + \varepsilon_t^i, \tag{6}$$

where i_t, π_t, and y_t represent the policy interest rate, inflation, and output, respectively, and ε_t^i is a monetary policy shock. The triplet (i^*, π^*, y^*) is typically interpreted as the steady state values for the variables.

In the literature, the second and third terms on the right-hand side of the Taylor rule are meant to be the cyclical response of the policy rate described in Figure 2(b) and Figure 2(c). They represent the attempt by the monetary authority to stabilize the equilibrium values of inflation and output around π^* and y^*. Most of the literature uses a variation of this Taylor rule, in which the triplet (i^*, π^*, y^*) is assumed to be time-invariant.[12]

Our interpretation of changes in the policy regime amounts to allowing for a target for inflation, denoted by π_t^*, that changes over time. These changes ought to be accompanied by the corresponding changes in the target for the interest rate, as implied by the Fisher equation in a steady state. So the value for the interest rate target, i_t^*, must also be time varying.

Our simple flexible prices quantity theory model has no bearing on interest rate movements that correspond to the second and third terms in the Taylor rule.[13] This is so much so that the implied relationship between the nominal interest rate and inflation in our model—as described, for instance, in (4)—is positive and one to one. In contrast, the conventional wisdom in central banks, supported by the workings of New Keynesian

[11] The "tightening" cycles are followed by their corresponding "easing" period, in which the interest rate is decreasing.

[12] For exceptions, see Ireland (2007), Cogley and Sbordone (2008), Ascari and Sbordone (2014) and Uribe (2022).

[13] In some formulations, such as that of Woodford (2003), the term $(y_t - y^*)$ in the Taylor rule is the difference between the equilibrium value for output and the one that would prevail under flexible prices – the output gap. In our simple quantity theory model, the output gap is by definition zero, so even that term disappears from the rule.

models, is that increases in the nominal interest rate imply *reductions* of inflation.[14]

In deciding the best value for λ, we aim at capturing the slow-moving term i_t^*, while we expect the filter to remove the second and third terms in the rule, as well as temporary stochastic disturbances.

This discussion explains our criterion for choosing the value for λ: the smallest value that eliminates from the data the tightening cycles. In Figure 2(b) and Figure 2(c), we show the effect of two alternative values for the trend component, corresponding to values of 6.5 and 100. The first value, 6.5, is the one that the RBC literature suggests for yearly data, and the result is shown in Figure 2(b). Its object of study is very different from ours (note the R in RBC), so there is no reason why what fits its objective should fit ours. As can be seen in the figure, it does not: when using $\lambda = 6.5$, the tightening cycles are still visible. On the other hand, when using $\lambda = 100$, the cycles are completely removed from the policy rate, as shown in Figure 2(c).[15] In what follows, we set $\lambda = 100$. In Appendix B.3, we also show the results when using $\lambda = 6.5$.

Conventional wisdom states that to see the mechanics of the quantity theory operating in the data for countries like the United States, one needs to look at averages over decades. This piece of conventional wisdom is consistent with Milton Friedman's own view of the lags in monetary policy. For instance, in his 1970 Wincott Memorial Lecture, delivered at the University of London, he wrote, *"In the short run, which may be as much as five or ten years, monetary changes affect primarily output. Over decades, on the other hand, the rate of monetary growth affects primarily prices."*

The particular value for λ gives some indication of the length of time over which a regime change can be identified. The average tightening cycle implied by the cyclical component of the interest rate is about three and a half years, with a maximum of six years in the late '90s and a minimum of one year, in 1967. And as Figure 2(c) makes clear, the filter with a value for $\lambda = 100$ totally removes all the tightening cycles. One could therefore interpret the filter as removing out of the data all fluctuations that last less than four years on average, the average duration of the tightening cycles in the United States. We adopt such an interpretation in the less formal discussion we engage in the concluding section.

[14]See Uribe (2022) for a masterful integration of these seemingly contradictory statements.

[15]The behavior of the trend component obtained for values of lambda between 90 and 110 are indistinguishable to the eye, given the size of these figures. How could we resist the seductive power of a round number like 100?

4 An Estimated Model with Monetary Regime Changes

Using filtered data allows us to avoid taking a stand on what drives short-run fluctuations of inflation. The advantage, as Lucas (1980) argued, is that this analysis can be applied to any theory in which the effect of frictions does not last more than four years on average. On the other hand, by taking a stand on what drives short-run fluctuations, we can estimate a specific model and search for a regime change that, in principle, makes the experience amenable to study the QTM implications. In addition, using model simulations, we can evaluate the ability of the filter to capture the regime changes.

Therefore, in this section, we move in the direction that Lucas (1980) discussed but did not pursue and estimate a standard small-scale New Keynesian model with two novelties. First, and in order to evaluate the first illustration, we do not consider the moneyless limit and use data on money in estimation. Second, and more importantly, we allow for policy regime changes that imply a time-varying inflation target in a way we make very specific below. We let the data speak about the timing and magnitude of these regime changes; in other words, we let the data inform us about how important this regime change is in explaining movements in inflation. For reasons of space and data availability, we focus only on the US. And for simplicity, we use a simple closed economy model in estimation. As we show below, we clearly identify a regime change that started with a gradual increase in the inflation target in the late '60s. A new regime change occurred with a gradual decrease in the inflation target in the early '80s.

Looking through the lens of the estimated model, we think of the data as being generated from different regimes – in our case, from regimes with different unconditional means for the inflation target. Any one regime is covariance stationary, and so oscillations of all frequencies are present. A change of the unconditional mean is a change of the zero frequency of the process. Applying the HP filter to time series data generated by two or more regimes with different means gives rise to a cyclical component that does not really belong to any one regime. So one should not interpret the trend component as literally belonging to a single regime; rather, our aim is to assess quantitatively the extent to which the trend component of our filter manages to capture the estimated monetary regime changes.

We use the estimated model to simulate data and filter them using the HP filter with the lambda parameter as calibrated in the previous section. We repeat the exercise in the same model but with the regime change shut. The comparison between the two exercises makes clear that the policy regime change is essential in explaining what appear as the most relevant movements in inflation, interest rates and money growth over the sample.

We repeat the exercise but vary the degree of price frictions. We show that the price frictions barely change the implications regarding the most relevant movements of inflation, interest rates, and money growth, which in all cases are explained by the policy regime shocks. We interpret these exercises as evidence that the strength of the price frictions in the model does not change the most relevant movements of the simulated data. We see all this evidence as a validation of our filtering choice, since it is the case that when applying the same filter to simulated data, the estimated regime change is well captured by the trend.

We now briefly describe the model, discuss the estimation strategy and present the results. A full description of the estimation and a detailed analysis of the simulation exercises are provided in Appendix C.

4.1 The model

For the analysis that follows, we extend the simple model briefly discussed in Section 2 and fully developed in Appendix A to include sticky prices, endogenous leisure, and preference, markup and monetary policy shocks—essentially the workhorse New Keynesian model used by Ireland (2004), complemented with a real money demand function.[16] Our main point of departure is to allow for a regime change to the inflation target.

To estimate the model, we need to take a stand on how monetary policy is conducted. In theory, the evolution of the most relevant part of inflation depends on the monetary policy regime, but not on the instrument used to implement such a regime. This is also the case in practice, since we estimate the model using an interest rate rule first and a money rule second. The estimated policy regime change is essentially the same one, independent of the policy instrument used.

We will discuss in detail the model in which policy is specified by means of an interest rate rule, which is the standard procedure. We present the details of the model with a money rule in Appendix D. In discussing the results, we will also focus on the case of the interest rate rule benchmark, and for purposes of comparison, we also show some results under the money rule.

The model is composed of the familiar Euler equation (which is the Fisher equation of our simple model of Section 2), the New Keynesian Phillips curve, and the Taylor rule

[16]Details of the non-linear model can be found in Ireland (2004).

shown below:

$$
\begin{aligned}
x_t &= (z - \ln\beta) - (i_t - \mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t x_{t+1} + (1-\omega)(1-\rho_a)a_t && (7)\\
\pi_t &= (1-\beta)\pi^s + \beta\mathbb{E}_t\pi_{t+1} + \psi x_t - e_t && (8)\\
i_t &= i_t^* + \rho_i\left(i_{t-1} - i_{t-1}^*\right) + \phi_\pi\left(\pi_t - \pi_t^*\right) + \phi_x x_t + \varepsilon_{i,t}. && (9)
\end{aligned}
$$

In the equations above, x_t is the output gap, π_t is the log of the gross rate of inflation, and i_t is the log of the gross nominal interest rate.

We differ from Ireland (2004) in that we allow for the inflation target, π_t^*, and the corresponding target for the nominal interest rate i_t^* to depend on time, as shown in equation (9). Specifically, we assume that

$$
\begin{aligned}
\pi_t^* &= (1-\rho_\pi)\pi^s + \rho_\pi\pi_{t-1}^* + \mathbb{I}^s\varepsilon_{\pi^*,t} && (10)\\
i_t^* &= z - \ln\beta + \pi_t^*. && (11)
\end{aligned}
$$

According to equation (11), the implied target for the nominal interest rate, i_t^*, is determined by the steady state real interest rate, $z - \ln\beta$, and the inflation target, π_t^*. The variable z is the steady state growth rate of labor-augmenting productivity, which follows in logs a unit root with drift z, and β is the household's discount factor. The inflation target, π_t^*, follows a regime-dependent AR(1) process in which $\mathbb{I}^s$ is an indicator variable that is turned on at T^{on} and then turned off at T^{off}; that is,

$$
\mathbb{I}^s = \begin{cases} 1 & \text{for } t \in [T^{\text{on}}, T^{\text{off}}) \\ 0 & \text{otherwise.} \end{cases} \qquad (12)
$$

At the start of the sample (1960Q1), we set $\mathbb{I}^s = 0$ and $\pi^s = 0.005$, which is equivalent to an inflation target of annually 2%. Before T^{on}, shocks to the inflation target are turned off, and the model is a standard New Keynesian model with a constant inflation target.

At T^{on}, the inflation target changes in two ways. First, $\mathbb{I}^s = 1$, so $\varepsilon_{\pi^*,t}$ now affects the inflation target π_t^*. Second, we allow in estimation, but do not require, the long-run inflation target – that is, π^s – to change from $\pi^s = 0.005$ to $\pi^s = 0.005 + \Delta_\pi$, and Δ_π is to be estimated. Thus, at time T^{on}, the inflation target is subject to a permanent shock, Δ_π, and to persistent but temporary shocks, $\varepsilon_{\pi^*,t}$, until T^{off}. Notice that because of the persistence of the inflation target process, ρ_π in equation (10), the long-run inflation target is reached gradually. Finally, at time T^{off}, policy reverts to its original regime; that is, $\mathbb{I}^s = 0$ and $\pi^s = 0.005$. Figure 3 presents a graphic illustration of the process of π_t^* as in equation (10) without stochastic shocks $\varepsilon_{\pi^*,t}$.

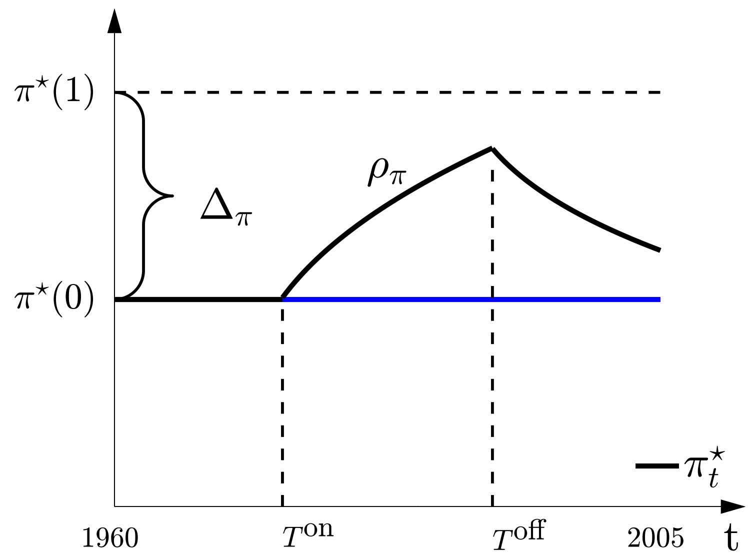


Figure 3: Graphic Illustration of Inflation Target Evolution

This choice allows for a potentially slow-moving component that pushes up inflation, with a reversion to the original 2% per year inflation rate. In estimating the model, we let the data choose the values for the five key parameters, $\{\Delta_\pi, \rho_\pi, T^{\text{on}}, T^{\text{off}}, \sigma_{\varepsilon_\pi}\}$, where σ_{ε_π} is the standard deviation of the shock $\varepsilon_{\pi^*,t}$ in equation (10).

We would like to highlight that there are two differences between the inflation target in regime 1 and the target in regime 0. First, as discussed above, there is a different long-run value for the inflation target. Second, there is a stochastic component that is absent in regime 0. We allow for this stochastic component only during regime 1 to capture the higher volatility of inflation that is apparent in the data when inflation is going up. In this way, this volatility does not need to be captured by a larger variance of the other structural shocks, although in estimation, we allow for that possibility, as we discuss in more detail below. We believe this to be the most reasonable way to model the regime change. Our results do not depend on that particular feature. We estimated the same model but without the stochastic component in regime 1, and the main results go through. For details, see Appendix D.

Notice also that we are keeping the slope parameters of the Taylor rule constant across regimes. This is an important assumption, since it has been argued—see for instance, Clarida, Galí, and Gertler (2000)—that the lack of a strong response of the policy rate to inflation in the '70s could explain the high inflation during the period. Thus, allowing for such a change could reduce the role of the inflation target shock. As we show in Appendix

D, this is not the case. In a nutshell, while the estimated slope coefficient on inflation is smaller during regime 1, the main results of this section do not change much. We chose to present in the main text the model in which those parameters are time invariant so as not to increase the number of estimated parameters too much.

We are not the first to consider a time-varying inflation target. But our approach of modeling regime changes in the inflation target is different from others proposed in the literature. In the Markov-switching (MS) specification proposed in Liu, Waggoner, and Zha (2011), the inflation target follows a Markov-switching process in which the target jumps between a high and a low value with non-absorbing transition probabilities. Although the inflation target in the policy rule changes, these changes are not permanent. As a result, the MS model is consistent with an ergodic (steady state) rate of inflation. In the MS model, agents expect the regime to switch with some probability in every period, but long-run inflation and long-run inflation expectations stay anchored at the ergodic mean. An advantage of our approach is that the steady state of nominal variables can shift. When a change in π^s takes place, long-run inflation and long-run inflation expectations adjust. In our case, there are two permanent shocks to the inflation target, Δ_π when the economy shifts towards the high inflation regime and then $-\Delta_\pi$ when the economy reverts back to the 2% regime. The target, π_t^*, adjusts gradually, as shown in Figure 3, towards its new long-run value.

Our approach is also different from that of Ireland (2007) and Uribe (2022), in which there are no regime changes, but the single regime is characterized by permanent shocks to the target in every period. This distinction will be key in our model simulations. Our posterior estimates imply only positive values for the parameter Δ_π and very high value for the persistence parameter ρ_π. Thus, in simulating the model, as we do in Section 4.3, the distributions of the simulations will be centered on an inverse U-shaped function, as it is in the data. Had we assumed permanent shocks, the simulations would all be centered on a flat line given by the sum of the initial condition of the permanent component and the mean of the transitory one. We will further discuss this in Section 4.3.

The economy is subject to the following non-policy shocks: a preference shock, a_t; a markup shock, e_t; a money demand shock, ξ_t; and a technology shock, z_t. These are governed by the equations below:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t} \tag{13}$$

$$e_t = \rho_e e_{t-1} + \varepsilon_{e,t} \tag{14}$$

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{m,t} \tag{15}$$

$$z_t = z + \varepsilon_{z,t}. \tag{16}$$

Real money demand m_t follows

$$m_t = \bar{m} + \rho_m m_{t-1} - (1-\rho_m)\eta\left(\frac{1+i^s}{i^s}\right) i_t + \xi_t. \tag{17}$$

4.2 Estimation

4.2.1 Estimation strategy

We estimate the dates of regime change, T^{on} and T^{off}, alongside the structural parameters, following the method outlined by Kulish and Pagan (2017). The model is estimated with five observable series: real GDP per capita growth; the federal funds rate; core inflation as measured by the CPI, excluding food and energy; the Michigan survey measure of inflation expectations; and money growth. For the United States, as discussed above, we use NewM1, the monetary aggregate proposed in Lucas and Nicolini (2015).[17]

The equations linking the observable variables; output growth, g_t; and money growth, μ_t, to the endogenous variables are given by

$$g_t = \hat{y}_t - \hat{y}_{t-1} + z_t \tag{18}$$

$$x_t = \hat{y}_t - \omega a_t \tag{19}$$

$$\mu_t = m_t - m_{t-1} + \pi_t + g_t \tag{20}$$

$$\mathbb{E}_t^{\text{obs}}\pi_{t+1} = \frac{1}{4}\left(\sum_{j=1}^{4}\mathbb{E}_t\pi_{t+j}\right) + v_t, \tag{21}$$

where $\hat{y}_t$ is the percentage deviation of stochastically detrended output, Y_t/Z_t, from its steady state; and $m_t = \ln(M_t/P_tY_t)$ is the log of real money balances to output. The constant $\bar{m}$ pins down real money balances to output in steady state. We use the Surveys of Consumers from the University of Michigan as the measure of inflation expectations, $\mathbb{E}_t^{\text{obs}}\pi_{t+1}$, and allow for measurement error, v_t.

In the steady state, $\pi_t = \pi_t^* = \pi^s$, $i_t = i^s$, $g_t = z$, and $i^s = \pi^s + z - \ln\beta$; all other variables (including the output gap) settle on zero. The reason nominal variables are left in levels, as opposed to percentage deviations from the steady state, is that in estimation we allow for changes in the steady states of these variables.

We estimated the model treating the regime changes as unanticipated. This seems to us a reasonable choice, particularly for the shock T^{on}: it is conceivable that the breakdown of the Bretton Woods system and the inflation that ensued took most by surprise. It is less

[17] The same results are obtained if we use the cash component of M1, which was not much affected by the regulatory changes on the '80s, as can be seen in the top-left panel of Figure 8.

plausible that the disinflation shock, T^{off}, was a surprise. However, we allow the change in target to be very slow moving by allowing for the autoregressive component in (10). A high value for ρ_π implies that the economy slowly approaches the new long-run target. The estimation does deliver a very high value for ρ_π, so although Δ_π is unanticipated, the transition path that it triggers for π_t^* towards its new steady state is anticipated.

To guard against the possibility that our proposed policy regime change captures the higher macroeconomic volatility before the Great Moderation, we use a parsimonious specification and introduce the parameter κ, which multiplies the standard deviations of all structural shocks, except that of money demand, before T_κ. That is, the standard deviation of structural shock i is given by $\kappa\sigma_i$ before T_κ and shifts to σ_i at T_κ. The standard deviations of all shocks change by the same proportions. By adding this "great moderation" shock, the estimation is free to rely on shocks other than the inflation target shock to account for the increased volatility in the earlier part of the sample.

The parameters that determine the steady state of output growth, the nominal interest rate, inflation and the ratio of money to output are set prior to estimation. In particular, we set $\beta = 0.9975$, $z = 0.0044$, $\pi^s = 0.005$ and $\bar{m} = 1$. Jointly, they imply a mean growth rate of real GDP per capita of 1.8% in annual terms, a mean nominal interest rate of 4.75%, an inflation rate of 2% in annual terms, and a ratio of money to output of about 25% in annual terms. We set the slope of the NK Phillips curve to $\psi = 0.3$, which in our model depends on a quadratic price adjustment cost. One way to interpret this slope is by considering a version of the NK model with a Calvo price friction. With log-utility and linearity in hours worked, a value of $\psi = 0.3$ in the standard NK model would correspond to a parameter ζ of 0.6, consistent with the findings of Fitzgerald, Jones, Kulish, and Nicolini (2020).[18]

4.2.2 The regime change

The two parameters characterizing the policy regime change are the persistence parameter ρ_π and the change in inflation target between regimes Δ_π. The prior on ρ_π is a beta distribution with a mean of 0.5 and a standard deviation of 0.2. The estimated posterior mode for ρ_π is 0.98, implying a very slow adjustment of the target to its newer, higher value. We use a wide uniform prior for Δ_π that ranges from -8% to 24% in annual terms. At the mode, Δ_π is estimated at roughly 0.01, which in annual terms amounts to a jump in the target of about 4% per year.

For the date breaks, T^{on} and T^{off}, we use uniform priors but restrict T^{off} to lie between 1979Q4 and 1983Q4, the quarters corresponding to the Volcker disinflation. In turn, T^{on} is

[18] ζ refers to the fraction of sticky firms.

restricted to take place simply before 1979Q4. Importantly, while the estimation allows for changes in the policy regime, these changes are not imposed. The estimation is free to choose $\Delta_\pi = 0$ and $\sigma_\pi = 0$, if it so desires.

The data strongly favor a specification in which the increase in inflation in the '70s is in large part interpreted as permanent, with π^s smoothly increasing from 2% to roughly 6% at the mode and with negligible mass for $\Delta_\pi < 0$. Most of the remaining variation is explained by temporary shocks to the inflation target. The date breaks are precisely estimated, with the inflationary regime beginning in the late '60s and ending in the early '80s. The estimates of the policy rule parameters are in line with those found in the literature. In the interest of space, the full set of estimates of the structural parameters and date breaks appears in Appendix C.

In terms of the time-varying size of the shocks, we estimate a value for κ of 2 at the mode, which implies that the volatility of structural shocks halved after T_κ, which at the mode is estimated precisely around 1985Q1.

The main difference across regimes is that when $\mathbb{I}^s = 0$, the inflation target shocks are shut down, and once $\mathbb{I}^s = 1$, shocks to the inflation target can have an impact on endogenous variables. To gauge how the contribution of structural shocks changes with the policy regime to inflation fluctuations, we conduct the following two exercises.

In our first exercise, we simulate the estimated model, setting the policy target shocks equal to their estimated values while setting the value for all other shocks to zero, as shown in Figure 4(a). In our second exercise, we repeat the exercise but set the policy target shocks to zero and set all other shocks to their estimated values, as shown in Figure 4(b). We conduct the two exercises for the model with an interest rate rule and also the model with a money rule. We show the corresponding counterfactuals as solid black and dashed blue lines in Figure 4(a) and Figure 4(b), respectively. For the ease of comparison, we also show the values for inflation in the United States during the period. Figure 4(a) makes clear that, independent of the monetary policy instrument we specify, the shocks to the target alone can do a very good job at tracking the evolution of the long-run component of inflation in the data. All other shocks only add to short-run inflation fluctuations but fail in capturing the high inflation from mid '60s to early '90s, as shown in Figure 4(b).

We further compare variance decompositions for Regime 1 and Regime 2 to understand how the contribution of structural shocks to other variables changes with the policy regime.[19] Table 1 presents variance decomposition results for each estimation under an interest rate

[19]This decomposition of the unconditional variance is due to the structural shocks alone, capturing what the unconditional variance would be if the regime were to prevail indefinitely. It does not account for the fraction of the variance in the data that results from permanent changes of the inflation target, from $\Delta\pi$.

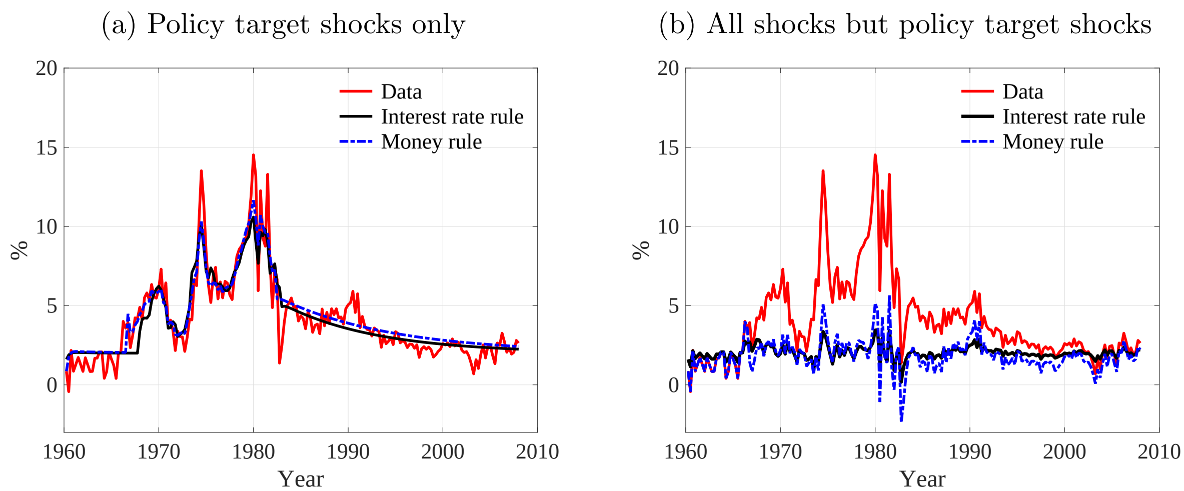


Figure 4: Counterfactual Simulations of Inflation with and without Regime Changes

rule and a money rule. Panel A of Table 1 shows that shocks to the inflation target, ε_{π^*}, account for the bulk of fluctuations in inflation and the nominal interest rate in Regime 2. Interestingly, the variance decomposition for real GDP growth, g_t, is essentially the same for the two regimes, with productivity shocks accounting for around three-fourths of its variance across regimes and the target shocks accounting for just 0.5% of its volatility.

Thus, not accounting for these monetary policy regime changes will wrongly assign these fluctuations to other shocks and lead to biases in the estimates. Panel B of Table 1 further shows that the same pattern emerges in the model with a money rule as well and that policy shocks explain most of the variation in inflation and interest rates in Regime 2.

Note also that in Regime 2, most of the volatility of both nominal interest rates and inflation is explained by the common target shock. By contrast, in Regime 1, it is the preference shock that explains interest rates, while the markup shock explains inflation. One interpretation of the results is that by imposing a constant target, the econometrician is ruling out the possibility of the common policy shock that is implied by the Fisher equation and that is evident in cross country comparisons.

These results are in contrast with findings of Galí and Gertler (1999) and Smets and Wouters (2007), who assume a fixed inflation target and find that wage and price markup shocks play dominant roles in explaining inflation fluctuations. Without a regime change of the inflation target, they find that monetary policy shocks explain a small part of federal funds rate's fluctuations and account for only a little part of inflation fluctuations.

Our results are also in contrast with the MS model of Liu, Waggoner, and Zha (2011), who ascribe the rise and fall of inflation more to shocks than to changes in the monetary policy regime. In their case, the high inflation target regime and the probability of switching

Table 1: Variance Decomposition

	Regime 1: $\mathbb{I}^s = 0$					Regime 2: $\mathbb{I}^s = 1$				
	i_t	π_t	g_t	μ_t	x_t	i_t	π_t	g_t	μ_t	x_t
Panel A: interest rate rule										
ε_i	2.2	6.2	4.0	3.7	22.6	0.4	0.8	4.0	2.5	21.2
ε_a	94.5	15.6	23.2	19.4	70.8	18.1	1.9	23.1	11.1	66.5
ε_e	3.3	78.2	0.4	1.2	6.6	0.6	9.7	0.4	1.6	6.2
ε_z	0.0	0.0	72.4	20.7	0.0	0.0	0.0	72.0	18.6	0.0
ε_{π^*}	0.0	0.0	0.0	0.0	0.0	80.8	87.6	0.5	16.6	6.0
ε_m	0.0	0.0	0.0	55.0	0.0	0.0	0.0	0.0	49.5	0.0
	Regime 1: $\mathbb{I}^s = 0$					Regime 2: $\mathbb{I}^s = 1$				
	i_t	π_t	g_t	μ_t	x_t	i_t	π_t	g_t	μ_t	x_t
Panel B: money rule										
ε_μ	0.2	3.0	1.2	7.2	10.6	0.2	0.2	1.9	4.7	14.5
ε_a	97.3	16.8	20.3	55.6	54.2	33.9	0.2	11.1	9.0	10.4
ε_e	0.8	72.2	0.2	3.5	4.5	0.6	6.0	0.3	1.8	5.9
ε_z	0.5	2.1	75.8	10.0	9.3	0.4	0.2	80.3	12.3	14.5
ε_{μ^*}	0.0	0.0	0.0	0.0	0.0	64.3	92.9	2.0	43.8	23.2
ε_m	1.1	5.9	2.5	23.7	21.3	0.7	0.5	4.5	28.4	31.4

to that regime explain some of the first increase in inflation of the mid '70s. But their counterfactual path under the permanently low inflation target tracks the actual path of inflation almost perfectly. As we argue above, their MS model is consistent with an unconditional mean (steady state) rate of inflation, while in our model, the unconditional mean of inflation *changes* with the monetary regime. This is a subtle but crucial difference. In our case, as can be seen in Figure 4(a), the lion's share of the rise and subsequent fall in inflation is explained by the change in regime. Our regime change involves a permanent change in inflation, and while the high inflation target regime is in place, $\mathbb{I}^s = 1$, transitory shocks to the inflation target explain over 85% of the variance of inflation in that regime.

Our model explains the data through a sequence of different monetary policy regimes. Because each regime is covariance stationary, inflation and other variables have well-defined moments within each regime. We rely on this fact when reporting the variance decompositions of Table 1. Our variance decompositions are conditional on each regime and therefore comparable to those reported by Smets and Wouters (2007). In the models of Ireland (2007) and Uribe (2022), however, inflation follows a unit root, so the variance of inflation is not well-defined. Their variance decompositions are not comparable to ours, because in Ireland (2007), they pertain to the *deviation* of inflation from the unit root inflation target, and in Uribe (2022), they pertain to the *first difference* of inflation.

Actual inflation in our sample is accounted for by both shocks and regime changes. To get a sense of how important regime changes are in accounting for actual inflation, we simulate a counterfactual economy under Δ_π and $\varepsilon_{\pi,t}$ but set all other shocks to zero. We take the ratio of the standard deviation of the inflation under the counterfactual to that of the data and find that the regime change accounts for 81% of the observed standard deviation.

4.3 Simulation analysis

We now use the estimated model to run several simulations that show the importance of the estimated regime change.[20] We also use simulations of the model to evaluate the ability of our proposed filter in capturing the estimated regime change.

First, we simulate the model 40 times, setting the estimated regime change and shocks to the target at the mode of the estimated posterior distribution and drawing all other shocks randomly from their posterior distributions.

We plot the 40 simulations, together with the inflation data on the left panel of Figure 5(a), and plot the corresponding filtered series on the right panel. In both cases, the data are represented with a wider black line. We next re-estimate the model but set all the shocks to the target to be zero, which is the standard procedure in the New Keynesian literature. We then simulate again the model 40 times, drawing all shocks from their posterior distributions. The left panel of Figure 5(b) shows the 40 simulations plus the data for the period, and the right panel shows the filtered version of the series in the left panel. As the figures make clear, the model without the shocks to the target cannot reproduce the trends detected in the data.

A model with permanent shocks to the inflation target would fail if shocks to the target were to be shut down. But it would also fail if one were to draw target shocks randomly given the unit root process for inflation. In our case, the deterministic component of the regime change would bring about a persistent increase in inflation even if the stochastic component were to be turned off. We also prefer our specification because it implies that post-Volcker, the inflation target remains constant at the stated objective of the Federal Reserve of 2% per year.

[20] In the text, we focus the analysis on the evolution of inflation. In Appendix C, we show the results for the nominal interest rates and for money growth.

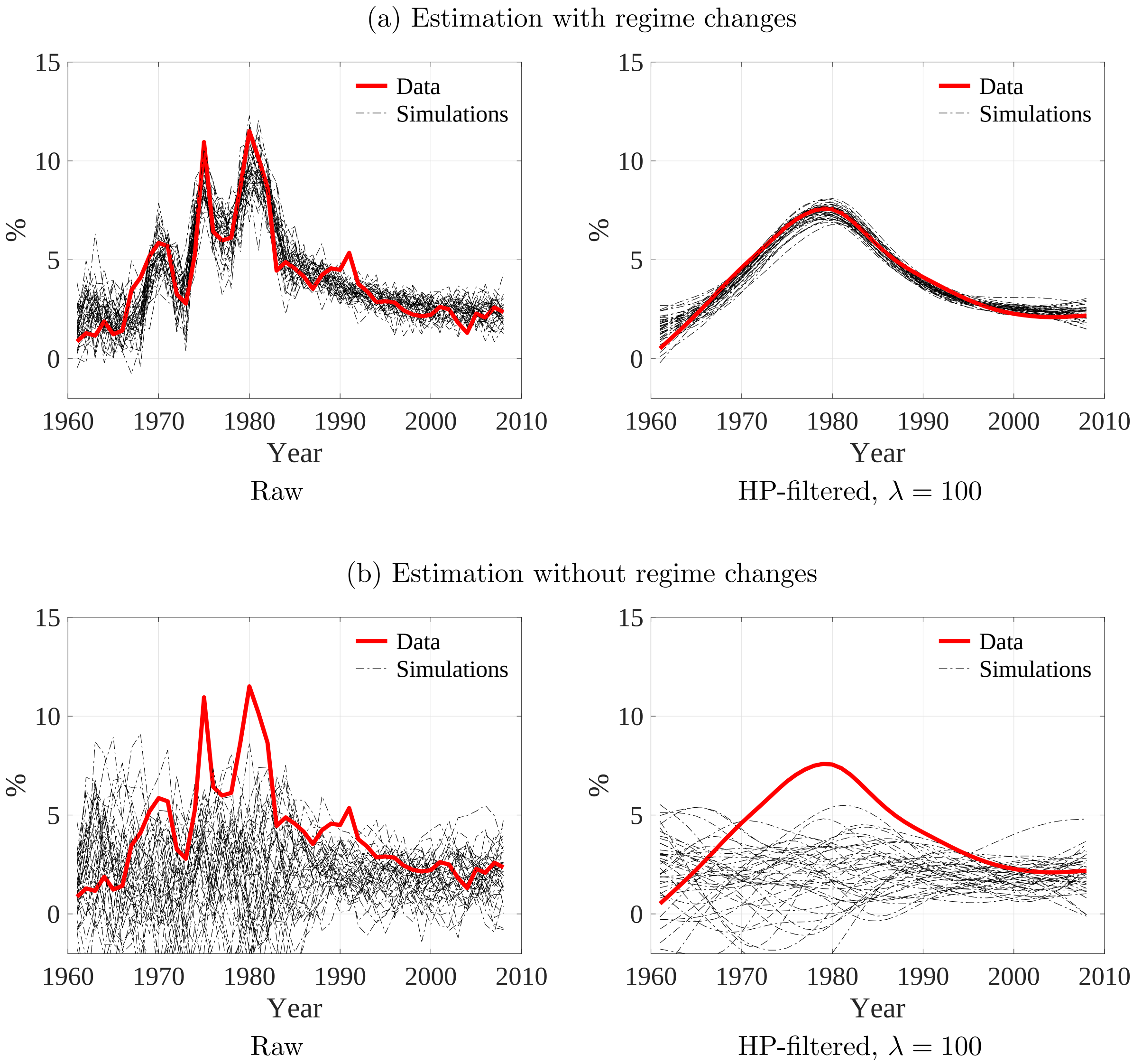


Figure 5: Model Fitness of Inflation Rates – Interest Rate Rule

The preceding exercises all point in the same direction: the most relevant movements in the data are well captured by the shocks to the target, while all the other shocks typically used in the literature have a very hard time accounting for them, even if we do not allow for the shocks to the target in the estimation. In addition, the filter does a very good job in tracking the evolution of the inflation target across regimes.

Sensitivity to price-setting frictions In order to evaluate the role of the price frictions, we simulate the model by setting all shocks to their estimated values but vary the value of Calvo parameter ζ at levels of 0.9, 0.6, and 0.1. Recall that we calibrate ζ to be 0.6 in the estimation, so that case corresponds to the true data. Then, we filter the simulated data and present the results for inflation in Figure 6.

(a) Simulation with regime changes (b) Simulation without regime changes

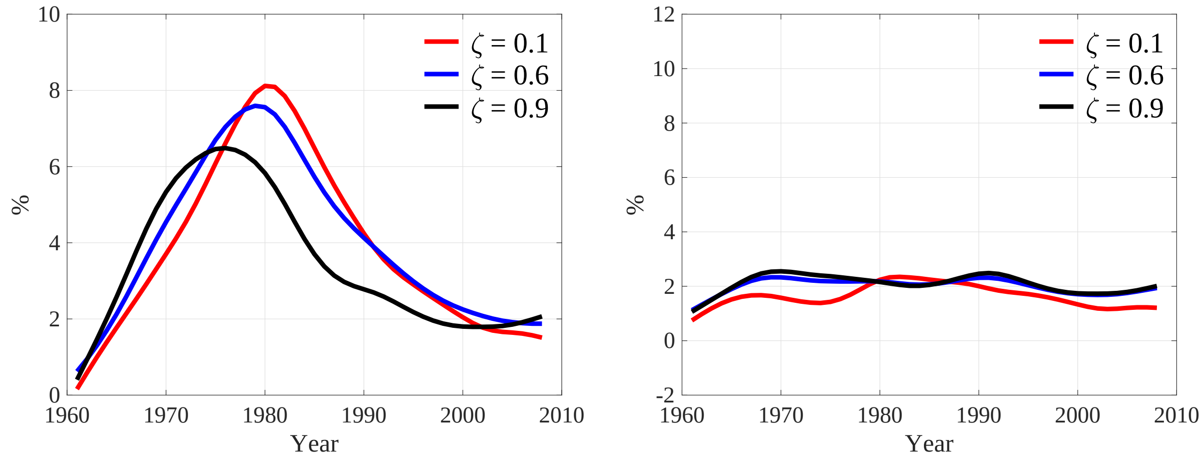


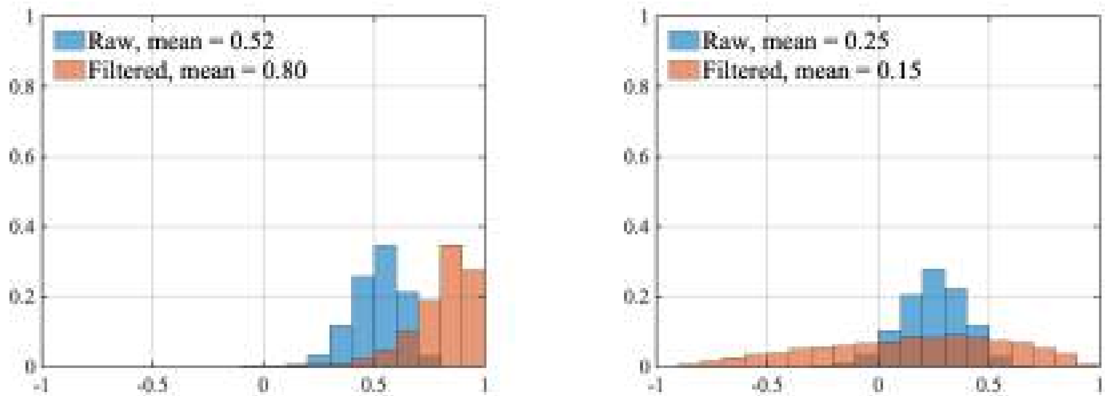
Figure 6: HP-Filtered Inflation Rates by Degree of Price Stickiness – Interest Rate Rule

As Figure 6(a) shows, while the specific value for ζ does change both the maximum inflation attained and the date at which it occurs, the differences are relatively small despite the large variation in values of ζ. In Figure 6(b), we report the results of the same exercise, with the values of the estimated shocks to the target equal to zero. As in Figure 6(a), the differences across regimes with different degrees of price friction are negligible, and in no case is it possible to reproduce the rise and subsequent fall in inflation that characterized the data in the United States between 1960 and 1990. These results reinforce the notion that the strength of the monetary transmission mechanism is not crucial for understanding the main trend observed in the inflation rates of the OECD countries presented in Figure 1.

Quantity theory correlations In the next section, we will apply our filter to the data to evaluate the two illustrations. Before doing so, we use simulated date to further evaluate the extent to which the existence of a regime change affects the performance of the filter.

We simulate the model 10,000 times, keeping fixed the regime change for the inflation target process at its estimated mode and drawing all other structural shocks from their estimated posterior distributions. In each simulation, we compute the correlation between the inflation rate and the two theoretically computed inflation rates — our two illustrations. We do so both for the simulated data and for the filtered version. We then compute the distribution of the correlation in both cases. The distributions obtained through this process are depicted in the left panels in Figure 7. The distributions of the correlation coefficients before and after the filtering in each case are depicted in blue and orange, respectively. Figures in panel (a) show the first illustration, while figures in panel (b) show the second. We also report means for all distributions.

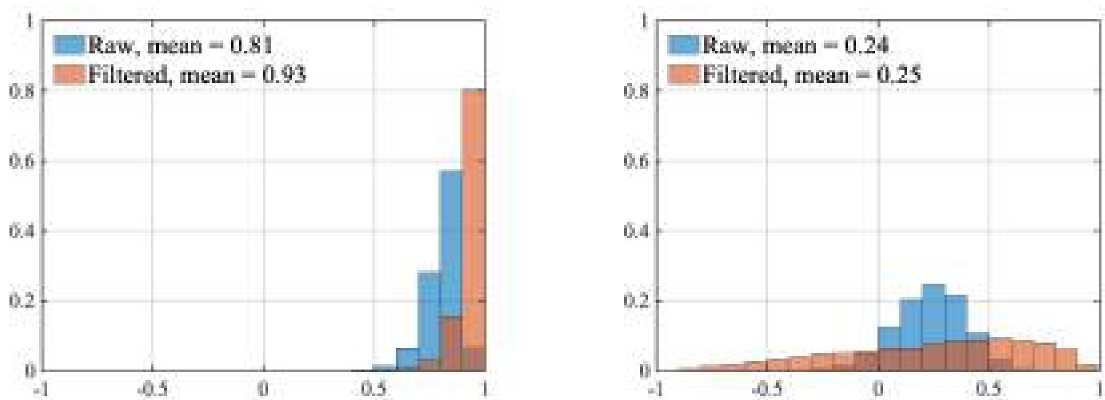
(a) Illustration 1



Benchmark with regime changes

Without regime changes

(b) Illustration 2



Benchmark with regime changes

Without regime changes

Figure 7: Correlations of Series in Simulated Data – Interest Rate Rule

As is evident, the distributions for the correlations are centered on positive values (0.52 for illustraion 1 and 0.81 for illustration 2). In addition, the correlations for the filtered data are centered on much higher values (0.81 and 0.93 respectively).

As a comparison, we repeat the exercise but set the shocks to the target equal to zero and draw all other shocks from their posterior distributions. We depict the distribution in right panels. Even though the two quantity theory predictions hold by construction in the model, the lack of shocks to the target implies that the correlations are lower in the simulated data. More importantly, filtering the data actually worsens the fit, in line with the two opening quotes of the paper. The filtered correlations with no regime change—that is, when there is no long-run variability—serve to highlight the point made by McCallum (1984) that low-frequency measures of the relationship between variables may be uninformative about their long-run relationship.

5 Two Illustrations for OECD Countries

We now take our two illustrations, equations (2) and (4), to the data. We selected countries that are members of the OECD and for which we have complete data since 1960. These are the 13 countries included in Figure 1, plus Colombia, Mexico and Turkey. These three countries experienced substantially higher inflation rates than the rest, so they have been left out of Figure 1.

Our strategy to validate the two illustrations is the same as the one in Lucas (1980). We filter the data and show the resulting plots for each country. We would like to emphasize that the strategy is consistent with the model estimated in Section 4, but it is also consistent with other alternative models with frictions that can rationalize short-term fluctuations whose effect vanish after a four-year period.

5.1 The data

We use the short-term interest rate on government debt for i, gross domestic product for output, and the CPI for prices. For the monetary aggregate, we use M1, which is the sum of currency plus checkable deposits. For the United States, M1 provides a misleading measure of total assets available for transactions, owing to regulatory changes that occurred in the early '80s. Lucas and Nicolini (2015) discuss this issue in detail and propose a new measure called NewM1, which adds the Money Market Demand Accounts (MMDAs) created in 1982 to the standard measure of M1. Thus, for the USA only, we use NewM1 rather than M1. Doing so raises the issue of whether the simple model described above could account for a regime change due to the regulatory changes in the middle of the sample. The model in Lucas and Nicolini (2015) does imply, not surprisingly, that such a regime change should change the relationship between the nominal interest rate and the ratio of money to output. Thus, for the USA only, we will also show the results using the currency component of M1, which according to Lucas and Nicolini (2015) should be relatively invariant to the regime change. In Appendix B, we discuss the data and their sources in detail.

As we mentioned above, we have no independent estimate for the real interest rate in the USA. Therefore, in the case of that country, we simply plot the inflation rate and the nominal interest rate, so as to appreciate the positive correlation. We then use the data to estimate an international real interest rate that we use to evaluate the second illustration in all the other countries.

The period we focus on is 1960-2005, as is consistent with the data in Figure 1. There are a few exceptions. For the countries that joined the eurozone, accurate measures of M1 are not available after 1999, since currency in circulation cannot be properly measured. For those countries, we use data up to 1999 in evaluating the performance of the money demand equation (2).

The presence of very low interest rates presents additional theoretical considerations that are worth discussing separately. The reason is that to obtain equation (2), we assumed a binding cash-in-advance constraint. The validity of that assumption at very low rates is questionable. Thus, for Japan, we end the sample in 1990, before the country lowered its interest rate to almost zero. In a final subsection, we separately discuss our analysis's policy implications for Japan since 1990, as well as the evidence since 2005 for other countries that experienced very low interest rates. Finally, because of data availability, we start the analysis of Turkey only in 1970.

5.2 Filtering results

In the top panels of Figure 8 to Figure 11, we show the data corresponding to the money demand equation (2) . We first plot the raw data for the inflation rate and for the growth rate of nominal money over real output. In the plots of raw data for Illustration 1, we do not make the adjustment for changes in the the nominal interest rate, as (2) implies. The reason is that this adjustment makes the theoretical prediction for inflation way more volatile than in the data, since the short-run movements in interest rates are very volatile and the value for the elasticity implied by the Baumol-Tobin formulation is high for data in the short run.[21] We then plot the trend component for the theoretical inflation, as predicted by equation (2), together with the trend component of inflation in the data. The bottom panels of Figure 8 to Figure 11 show the data corresponding to the Fisher equation (4). In all plots, we also report the correlation between the series.

The first column of Figure 8 presents the results for the United States. As mentioned above, in this case we use both cash and NewM1 for Illustration 1. The yearly data do not make apparent the relation between money growth and inflation when using New M1. In fact the correlation is just 0.18. However, once the long-run movements are isolated and the effect of changes in the interest rate is taken into account, as equation (2) implies, the

[21] This is consistent with the old empirical literature on money demand, which argued that the estimated "short-run" interest rate elasticity was much smaller than the "long-run" elasticity. See Lucas (1988) for a discussion.

Illustration 1

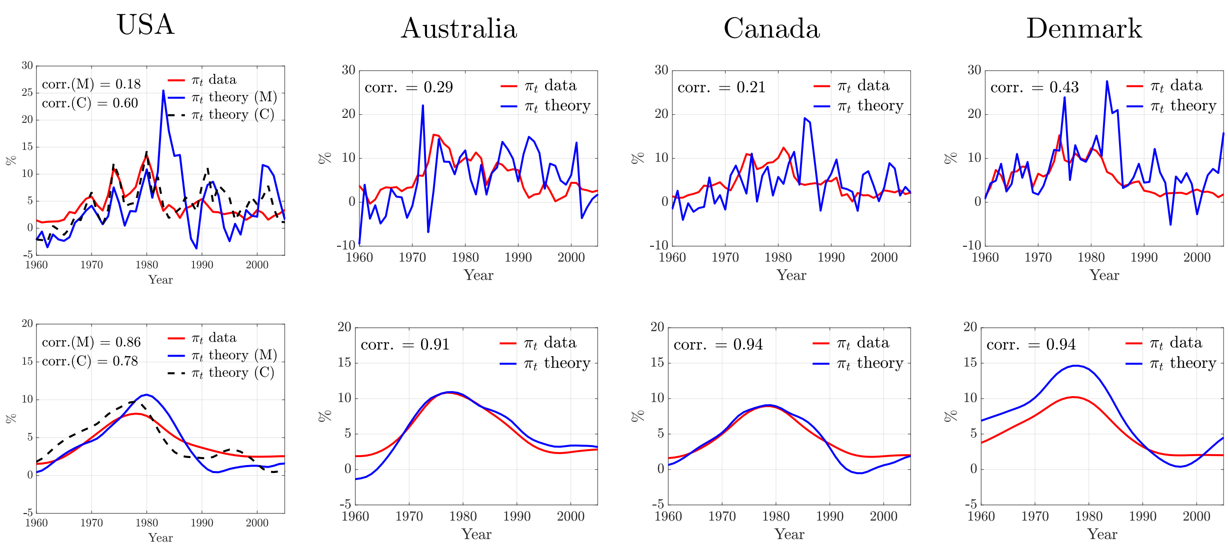


Illustration 2

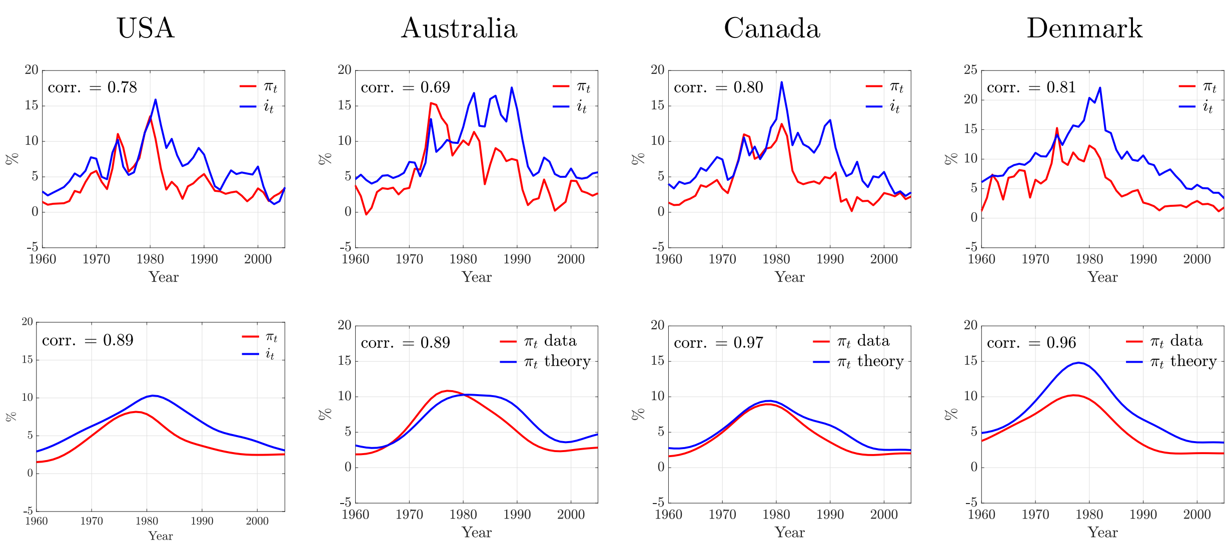


Figure 8: Countries in Group 1 (a)

Figure 9: Countries in Group 1 (b)

Illustration 1

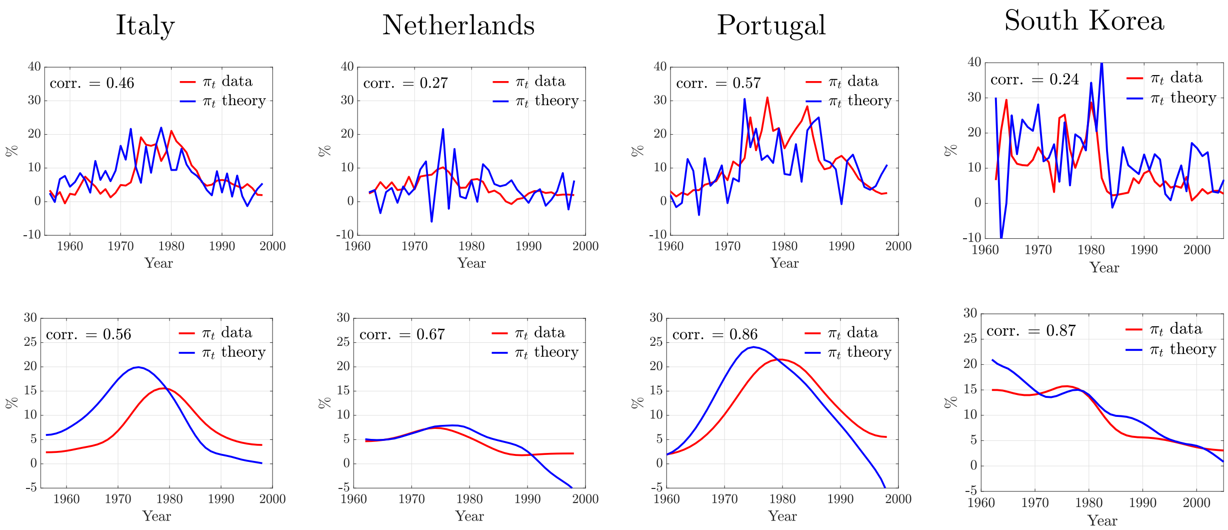


Illustration 2

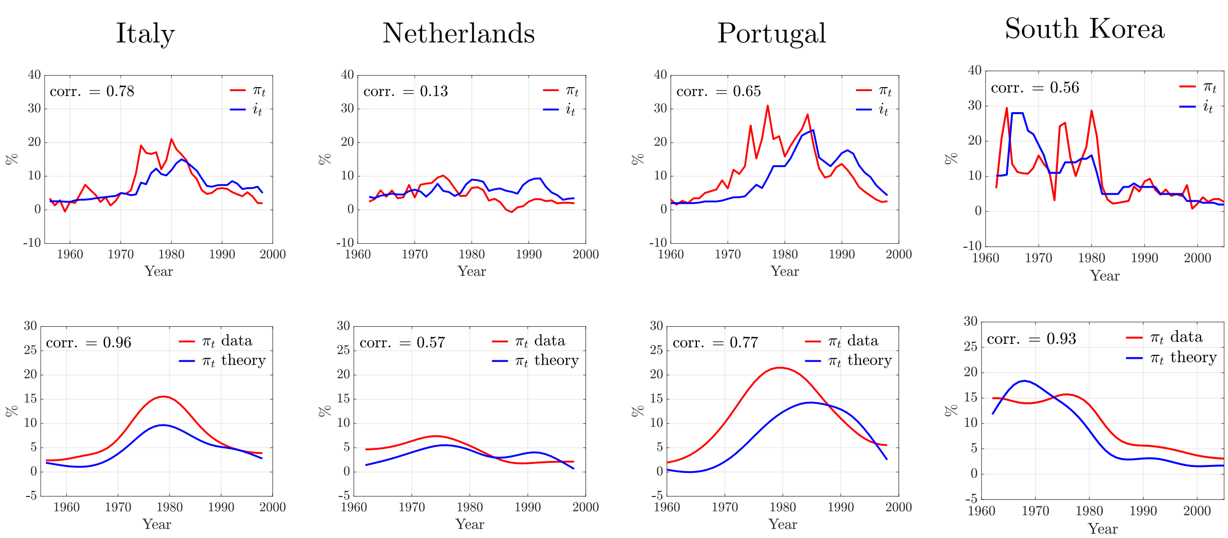


Figure 10: Countries in Group 2 (a)

Illustration 1

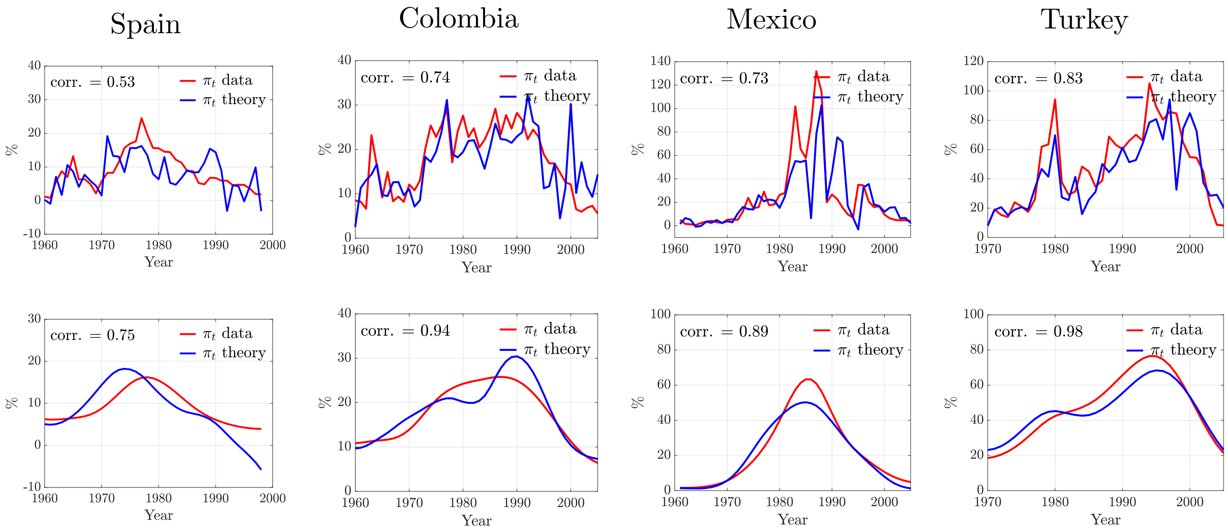


Illustration 2

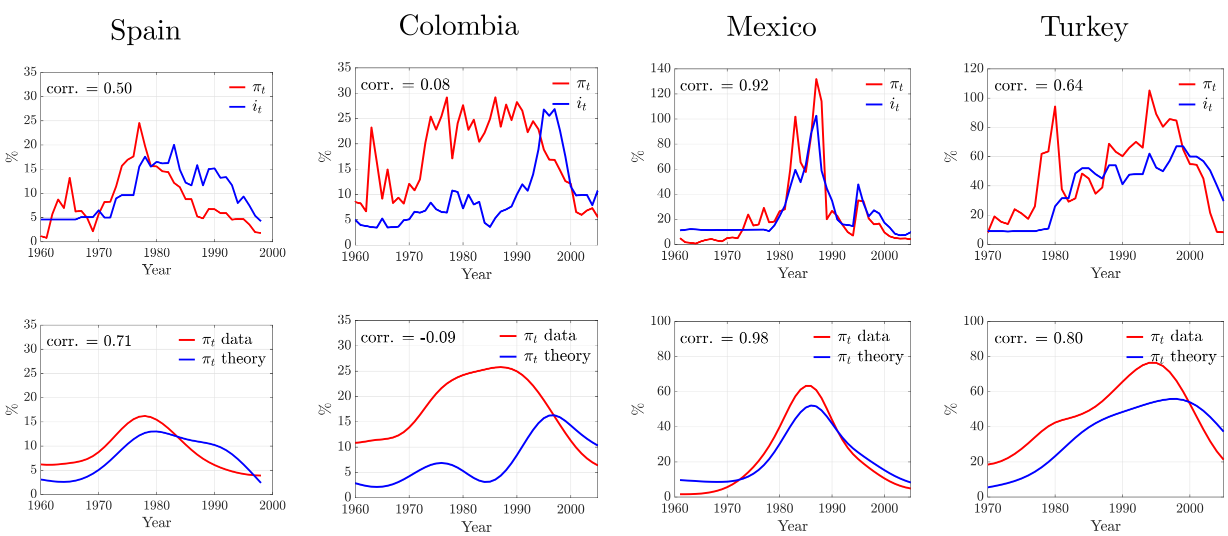


Figure 11: Countries in Group 2 (b)

match between the theory and the data is quite notable, with a correlation coefficient of 0.86 when using NewM1. This is in spite of the regulatory changes in the early '80s. The match when using just cash is much better in the data and almost as good when using only the trend component.

We separate the countries into two groups. The first group, shown in Figure 8 and Figure 9, includes the countries for which we do not find any particular behavior that makes our assumptions for the model especially suspicious. The worst fits in this group are Germany for Illustration 1, in which the correlation is only 0.35, and, to a lesser extent, the Netherlands. As we explained in Section 4.3, we interpret this failure as a lack of regime change for those countries. The second group, shown in Figure 10 and Figure 11, includes a set of countries for which the nominal interest rate is lower than the inflation rate for several years in the first two decades of the period analyzed. To us, this behavior suggests government intervention in the credit market, relatively common in the '60s and '70s and called "financial repression" at the time. Under this condition, the observed interest rate is not a market-determined price, so it does not represent the true opportunity cost of money. This imposes a bias in our two theoretical predictions. This group also includes cases with higher inflation rates.

Each picture is worth a thousand words. As we provide plenty of pictures, words will be kept to a minimum. We read the sequence of plots as an affirmation of our simple theory. We mostly let the readers evaluate the pictures themselves and emphasize just a few features of the plots.

First, while the correlation between the data and the theory is very high, in some cases there are sizable differences of up to a few percentage points; these do matter for policy. A 2% or larger difference between observed inflation and the theoretical counterpart, as observed in many cases, is an important difference that can and should be further studied on a case-by-case basis. It is most likely that in order to understand those differences, country-specific features ought to be brought to the policy debate table.

Second, for the group of countries in Figure 10 and Figure 11, for which we guess that financial repression was prevalent in the first decades of the sample, the evidence is worse, particularly when evaluating the second implication (the Fisher equation). A poster child of this issue is Colombia, where financial repression was the norm till the reforms of the early '90s.

5.3 The near-zero nominal interest rate periods

As is the case in cash-in-advance models like the one we solve in Appendix A, a positive interest rate is required for the cash-in-advance constraint to be binding. When this is not the case, real money demand is not uniquely determined. In our simple representative agent economy, the result is stark: as long as the interest rate is positive, the constraint is binding, and the equilibrium of the model is uniquely pinned down. However, sensitive modifications, like agent-specific borrowing limits or heterogeneous returns on nominal assets due to heterogeneous access to credit markets, would affect the implications of the model when the nominal interest rate is positive but very close to zero.[22]

To further clarify this discussion, consider equation (1) . Notice that the solution for real money balances as a fraction of output goes to infinity when the nominal interest rate goes to zero. How can that be a solution for agents that have finite wealth? The answer is that in equilibrium, the private sector's borrowing from the government is also going to infinity, keeping the wealth of the private sector bounded. While this is mathematically correct for any positive interest rate, it is of little, if any, applied interest.

To illustrate the difficulties in using real money demand theory at very low interest rates, we now analyze those countries that experienced them for a prolonged period of time. Besides the solution in (1) , we use an alternative functional form proposed by Selden (1956) and Latané (1960) and explored in detail in Benati et al. (2021). The specific functional form is given by

$$\frac{M_t}{P_t x_t} = \frac{A}{1 + b i_t}. \tag{22}$$

Notice that when $i_t = 0$, real money demand - as a fraction of output - is finite. Thus, it departs from the Baumol-Tobin specification at very low interest rates. On the other hand, the parameters A and b can be chosen so that (1) and (22) are very close to each other for interest rates above 2% and all the way up to 30%, a range that includes all the experiences we now discuss.

In Figure 12, we extend the analysis presented in Figure 8 and Figure 9 to several countries in Group 1 that maintained an interest rate very close to zero for several periods. As mentioned above, in computing theoretical inflation, we present both the log-log case used before and the Selden-Latané case, in which the parameters have been chosen to match as much as possible the solution in (1).[23]

As the figure shows, when using the log-log specification, the implications of money

[22]For an analysis with heterogeneous borrowing constraints, see Benati et al. (2021). An alternative model that delivers similar results is analyzed in Alvarez and Lippi (2009).

[23]Specifically, we choose parameter $b = 0.14$ for the Selden-Latané specification. The value for A is irrelevant for computing growth rates, as we do in this section.

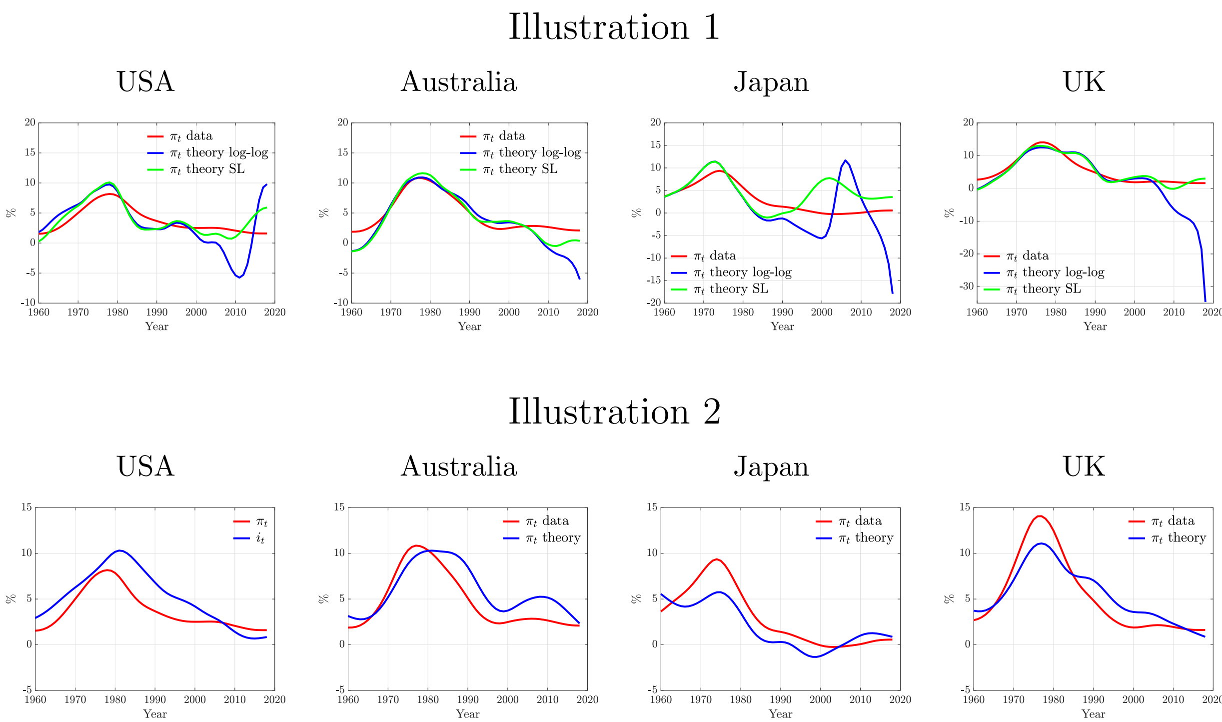


Figure 12: Illustrations for Countries with Periods of Low Interest Rates (1960–2018)

demand theory become really off the mark during the periods of very low interest rates. The Selden-Latané alternative specification does better but still fails to perform as it did in the previous years. Figure 12 suggests that inference about the behavior of real money demand at very low rates using evidence of periods with relatively higher rates, when the cash-in-advance constraint can be safely assumed to be uniformly binding, could be misleading. Thus, monetary aggregates may be uninformative at very low rates.

Note, on the other hand, that the evidence regarding the second illustration is as good for the low interest rate period as it is for the rest of the sample.

Two policy implications follow. First, when interest rates are very low, the effect of expansions of the central bank's balance sheet on the real side of the economy — the so called "unconventional policies" — is hard to predict, since it is hard to estimate the demand for those assets. Second, changing the trend component of the the policy rate in a way that resembles a positive shock to the target — as the Federal Reserve did between 2015 and 2018 — can act as an effective tool to fight persistently low inflation, even at the lower bound.

6 Conclusions: Policy Implications

Good day-to-day central banking is a complicated task: it amounts to monitoring and assessing massive amounts of data, simulating alternative scenarios, studying the robustness of policies in each scenario, and deciding how to weight each of these elements in the decision making process. These decisions affect the actions of many different members of society, none of whom know exactly how the economy functions. These actions in turn determine the way policy decisions affect economic outcomes. Given this complicated feedback, it is very tempting to disregard the lesson of very simple, almost naive theoretical constructions like the quantity theory of money.

We built a case for not falling into that temptation. The immediate effect of a monetary policy change depends on details of the environment, and relatively minor changes can sometimes substantially change the theoretical conclusions. But to understand inflation over relatively longer horizons, we argue that the quantity theory, though a simple and utterly unrealistic abstraction, suffices.

The estimation of a specific structural model, combined with simple filtering techniques, suggests that a reasonable definition of "longer horizon" is roughly four years. Given this, are there any direct policy implications that come out of our analysis?[24] We believe so, but it clearly depends on the question at hand. We illustrate this by addressing several very topical policy issues.

But before doing so, a caveat is in order: the analysis so far has focused on establishing a strong correlation between inflation and monetary policy variables in the medium term. The policy analysis that follows is based on two elements. The first is the strong evidence we presented above of co-movement between inflation, the nominal interest rates and money growth in the data, as predicted by the QTM, over a four year period. This evidence is complemented with the estimated model, where we very clearly identify a regime change in monetary policy as the main explanation of the high inflation of the seventies. The second is the assumption that we confidently adopt, which is that monetary policy can control either the short-term nominal interest rate or the growth rate of the money supply, at least in the medium term.

We start with one debate for which our analysis has nothing to contribute. Starting in mid 2022, the Federal Reserve has increased the policy rate at a speed not seen in decades. What will be its effect on the inflation rate for the first half of 2024? No useful answer can be derived from our analysis.

On the other hand, assume that the tightening cycle is followed by a return of the policy

[24] For a more detailed discussion of the policy implications of this analysis, see Gao and Nicolini (2023).

rate close to the current real rate of about 1.5% plus the 2% inflation target by 2025. This is four years after the inflation rate started to increase. Then, our analysis suggests that inflation should be close to the 2% target by that year, as long as only the usual business cycle shocks affect the US economy till then.

A corollary of that implication is that in the current fight to lower inflation in the short run, it is better to go for higher rates rather than to keep them high for longer. The latter case has the potential drawback of transforming a short-run increase in the policy rate into a change in the inflation target.

Our analysis can also be applied to the "low inflation" debate that prevailed in many developed countries during the pre-COVID years. Many central banks, including the Fed, lowered policy rates to almost zero following the 2008-09 financial crisis and maintained them at zero for a long period. That policy is consistent with a regime change with a lower inflation target, so these episodes could be interpreted as a deflation trap equilibrium, as described in Benhabib, Schmitt-Grohé, and Uribe (2001).

Under this interpretation, the low inflation observed during those years was the natural response to a change in the implied target for inflation. Other extreme measures were taken, in which the balance sheet of central banks ballooned, with no apparent response of inflation. According to the evidence presented here, this should not be very surprising given that the policy rates remained at zero for a very long period of time.

Incidentally, note that if this were the case, a policy of forward guidance, indicating that the policy rate will remain low for an extended period of time, could reinforce the notion that the interest rate is responding neither to the usual variables of interest nor to temporary events, and could therefore be interpreted as a drop in the inflation target.

Probably the most extreme case along this line is Japan, since it lowered the policy rate long before the other countries. Over the last two decades, the Bank of Japan has been very concerned about the low inflation rates in the country. This can be seen in Figure 13(a), which plots the trend component of inflation in Japan — the solid red line — together with the equivalent measure for the other seven countries in Group 1. The trends have all been obtained using the HP filter with a λ parameter equal to 100, as proposed in the paper. Japan appears as the clear outlier, with substantially lower inflation all the way till the end of the sample, at which point its inflation rate seems to converge with the group. Does the policy followed by the Bank of Japan explain this fact? We believe so. Figure 13(b) plots the trend components of the policy rates. Japan is again the clear outlier, with interest rates systematically lower than the rest, except at the end of the sample.

In the natural counterfactual in which Japan had maintained permanently higher interest rates — say, at the average value for the other countries—the inflation rate in

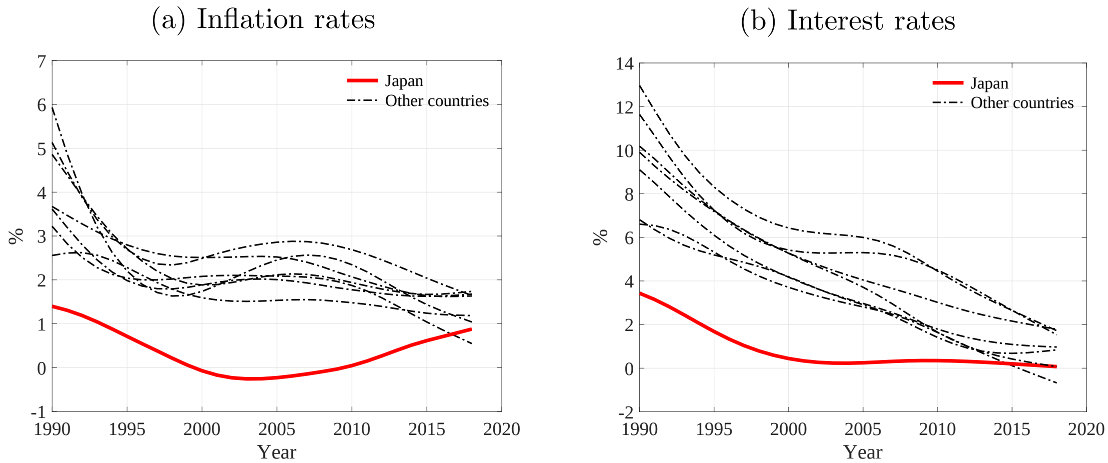


Figure 13: Low-Frequency Movements of Group 1 Countries since 1990

Japan would have also been higher—say, at the average value of the other countries.

The figure also hints at the reason why, over a decade ago, inflation in Japan started to increase somewhat. Notice that in Figure 13(a), the negative trend in nominal interest rates of all the other countries in Figure 13(b) has not been followed by the inflation rates in those same countries since the year 2000 or so. According to the model, this is possible only if the real interest rate also falls during the period by a magnitude similar to that of the fall in the nominal interest rates. This observation is consistent with the clear downward trend on inflation indexed bonds in the USA since their inception in the early 2000s, as we show in Appendix B. Under this interpretation, the marginal increase in inflation in the last decade in Japan is fully accounted for by the lower real rates that have been observed globally, since there is no movement in the trend component of the policy rate in Japan.

The behavior of the short-term interest rate in Japan suggests the possibility of a regime change, in which the target for inflation was substantially reduced at some point in the late '90s or early 2000s. This is not the official position of the Bank of Japan, but it is consistent with the estimation results in Jones, Kulish, and Nicolini (2023), which apply techniques similar to the ones developed in Section 4 to the case of Japan.

It turns out that Japan is also an exception among most OECD countries, in that the increase in inflation post-COVID has been the most moderate. And given that the policy rate has been kept at essentially zero, our analysis implies that as soon as the inflation burst of COVID disappears, and given the current positive real rates, deflation in Japan is likely to come back in the years to come if the low nominal interest rates policy is maintained.

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Online Appendix for

Two Illustrations of the Quantity Theory of Money Reloaded

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A An Open Economy Model with Money

A.1 Model setup

We consider a multi-country, two-sector model. There are J countries, each indexed by $j = 1, ..., J$ and the two sectors are tradable sector T and non-tradable sector N.

Households We assume that the period utility for a single country follows

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[u(c_t^N) + u(c_t^T)\right],$$

where c_t^N is consumption of a home good that is not traded in international markets, and c_t^T is consumption of a traded good. The function u satisfies standard Inada conditions.

To carry on transactions, households must pay with money, so we impose the following cash-in-advance constraint

$$M_t n_t \geq P_t^N c_t^N + P_t^T c_t^T, \tag{A1}$$

where n_t is the number of trips to the bank that the household makes every period. Making trips to the bank entails a fixed stochastic cost γ_t in units of time. The total time endowment is equal to 1, so feasibility requires

$$\gamma_t n_t + h_t + l_t = 1.$$

where h_t is the amount of labor devoted to the production of the non-traded good, and l_t is the amount of labor devoted to the production of the traded good.

Production Labor is the only input in production and technologies in country j are such that

$$\begin{aligned} y_{jt}^N &= z_{jt}^N h_{jt}, \\ y_{jt}^T &= z_{jt}^T l_{jt}, \end{aligned}$$

where z_{jt}^N and z_{jt}^T are stochastic productivity processes.

An interior solution for the production of both goods that equalizes wage W and marginal products of labor implies that

$$W_{jt} = z_{jt}^N P_{jt}^N = z_{jt}^T P_{jt}^T,$$

so

$$\frac{P_{jt}^N}{P_{jt}^T} = \frac{z_{jt}^T}{z_{jt}^N}.$$

For the non-trade good, consumption is equal to production in equilibrium, i.e.,

$$y_{jt}^N = z_{jt}^N h_{jt} = c_{jt}^N.$$

However, for the traded good, production is equal to consumption plus the trade balance in equilibrium. We denote the trade balance as τ_{jt}. We then have

$$y_{jt}^T = z_{jt}^T l_{jt} = c_{jt}^T + \tau_{jt}.$$

We also define the aggregate price level in country 1 as P_{jt} such that

$$P_{jt} c_{jt} = P_{jt}^N c_{jt}^N + P_{jt}^T c_{jt}^T.$$

Flow budget constraints in country 1 We assume that the currency of country 1 is used as the international currency, that we call the dollar. As country 1 is special relative to the others, we abbreviate from its country subscript when describing country 1's problem, and introduce a superscript $*$ to variables that are related to country 1 and enter other countries' problem, such as cross bonds holding and returns of the bonds.

Households in country 1 can hold a domestic bond, issued in dollars that pays a nominal return i_t, a bond that pays a real interest rate r_{t+1} and that is indexed to the prices, in fixed and equal proportions. Hereafter, we put on the time subscripts for prices and interest rates according to the time of realization. This explains our choice of notation that while both accrued within period t, nominal assets have return i_t, while real assets have return r_{t+1}. For the same reason, we denote the inflation as $\Pi_{t+1} = P_{t+1}/P_t$.

Households in country 1 can trade nominal bonds B_t, real bonds b_t, as well as a representative portfolio a_t as a collection of all other assets at price P_t^a. We want to explicitly allow for these assets, since we will use data on them in our empirical application. Technically speaking, to pin down the equilibrium, extra equilibrium conditions will be required for assets a_t. However, as in our first empirical section, we will not solve the full model, rather, we focus on two conditions that must hold in equilibrium, we do not need to specify the full set of assets available to households.[1]

Denote households' nominal wealth as Ω_t. Households in country 1 face the following flow budget constraints

$$B_t + P_t b_t + M_t + P_t^a a_t = \Omega_t \tag{A2}$$

$$\Omega_{t+1} = W_t(1 - \gamma_t n_t) - P_t^N c_t^N - P_t^T c_t^T + B_t(1 + i_t) + P_{t+1}(1 + r_{t+1}) b_t + M_t + P_{t+1}^a a_t \tag{A3}$$

together with the cash in advance constraint ($A1$). Without loss of generality, we only

[1] In our second empirical exercise, where we estimate the model, we do assume complete markets.

include one type of additional assets in the equation, which can be interpreted as a portfolio of all other assets and can also be easily extended to multiple types without affecting any of our following derivations.

Flow budget constraints in country j Households in country 1 do not hold bonds issued by other countries. However, households in other country $j = 2, ..., J$ can hold both nominal and real bonds that are available to households in country 1, each denoted as B^*_{jt} and b^*_{jt}. Additionally, they can also hold nominal and real bonds issued in their domestic currency, that pay a return i_{jt} and r_{jt}, respectively. As mentioned earlier, to distinguish between domestic and international bonds, we denote returns and prices of the international (country 1) bonds with $*$, i.e., $i^*_t = i_t, r^*_t = r_t$ and $P^*_t = P_t$.

Denote S_{jt} as the nominal exchange rate that transforms dollars into the domestic currency. Households in country j face the following constraints:

$$S_{jt}(B^*_{jt} + P^*_t b^*_{jt}) + B_{jt} + P_{jt}b_{jt} + M_{jt} + P^a_{jt}a_{jt} = \Omega_{jt} \tag{A4}$$

and

$$\begin{aligned} \Omega_{jt+1} &= W_{jt}(1 - \gamma_{jt}n_{jt}) + S_{jt+1}[B^*_{jt}(1 + i^*_t) + P^*_{t+1}(1 + r^*_{t+1})b^*_{jt}] \\ &+ B_{jt}(1 + i_{jt}) + P_{jt+1}(1 + r_{jt+1})b_{jt} + M_{jt} + P^a_{jt+1}a_{jt} - P^N_{jt}c^N_{jt} - P^T_{jt}c^T_{jt} \end{aligned} \tag{A5}$$

together with the cash in advance constraint ($A1$).

We also add trading constraints on assets from country 1, that can rationalize deviations from UIP, as the one studied by Itskhoki and Mukhin (2021)

$$f(B^*_{jt}) \leq 0, \quad \text{and} \quad g(b^*_{jt}) \leq 0.$$

As before, the assets a_{jt}, traded at price P^a_{jt}, are other assets available in country j, whose optimality conditions will be ignored.

A.2 The two illustrations in country 1

First order conditions The first order conditions with respect to consumption, bonds, money, and number of trips to the bank are given by

$$\beta^t u_{c^N}(c^N_t) = \lambda_t P^N_t + \delta_t P^N_t \tag{A6}$$

$$\beta^t u_{c^T}(c^T_t) = \lambda_t P^T_t + \delta_t P^T_t \tag{A7}$$

$$\phi_t = \lambda_t \mathbb{E}[\frac{P_{t+1}}{P_t}(1 + r_{t+1})] \tag{A8}$$

$$\phi_t = \lambda_t(1 + i_t) \tag{A9}$$

$$\phi_t = \lambda_t + \delta_t n_t \tag{A10}$$

$$\delta_t M_t = \lambda_t \gamma_t W_t \tag{A11}$$

where δ_t, ϕ_t, and λ_t are the multipliers on constraints ($A1$), ($A2$), and ($A3$), respectively.

Deriving the Fisher equation Combining equations ($A8$) and ($A9$) we obtain

$$(1+i_t)=\mathbb{E}[\frac{P_{t+1}}{P_t}(1+r_{t+1})].$$

Deriving the money demand To derive the equation for the money demand, it suffices to use equations ($A9$), ($A10$), and ($A11$), that are all static. Combining them, we can get

$$\frac{i_t}{n_t}M_t=\gamma_t W_t.$$

Assuming that the cash-in-advance constraint is binding, we get

$$\frac{i_t}{n_t}=n_t\gamma_t\frac{W_t}{P_t^N c_t^N+P_t^T c_t^T}.$$

As $W_t=P_t^N z_t^N$, $c_t^N=z_t^N h_t$, and $z_t^T l_t-\tau_t=c_t^T$, we obtain

$$i_t=(n_t)^2\gamma_t\frac{P_t^N z_t^N}{P_t^N z_t^N h_t+P_t^T\left(z_t^T l_t-\tau_t\right)}.$$

Since $P_t^T=P_t^N\frac{z_t^N}{z_t^T}$, it thus follows

$$i_t=(n_t)^2\gamma_t\frac{P_t^N z_t^N}{P_t^N z_t^N h_t+P_t^T\frac{z_t^N}{z_t^T}\left(z_t^T l_t-\tau_t\right)},$$

i.e.

$$i_t=(n_t)^2\gamma_t\frac{1}{h_t+l_t-\frac{\tau_t}{z_t^T}}=\frac{(n_t)^2\gamma_t}{1-\gamma_t n_t-\frac{\tau_t}{z_t^T}}.$$

The second equality holds because $\gamma_t n_t+h_t+l_t=1$. Notice that $\gamma_t n_t$ is the welfare cost of inflation, while $\frac{\tau_t}{z_t^T}$ is the trade balance in equivalent units of labor. Thus the sum $\gamma_t n_t+\frac{\tau_t}{z_t}$ is the fraction of output that is lost due to inflation, which has been estimated to be less than one percent of output, plus the value of the trade deficit over output. As these numbers are small, we approximate the denominator to be close to one and work with the approximation

$$\sqrt{\frac{i_t}{\gamma_t}}=n_t.$$

When the cash-in-advance constraint is binding, we then have

$$\frac{M_t}{P_t^T c_t^T+P_t^N c_t^N}=m_t=\frac{1}{n_t},$$

where m_t is the equilibrium quantity of money as a fraction of consumption.

A.3 The two illustrations in country j

The first order conditions with respect to bonds, money, and number of trips to the bank are given by

$$\phi_{jt} = \lambda_{jt}\mathbb{E}[\frac{P_{jt+1}}{P_{jt}}(1 + r_{jt+1})], \tag{A12}$$

$$\phi_{jt} = \lambda_{jt}(1 + i_{jt}), \tag{A13}$$

$$\phi_{jt} + \mu^b_{jt} = \lambda_{jt}\mathbb{E}[\frac{S_{jt+1}P^*_{t+1}}{S_{jt}P^*_t}(1 + r^*_{t+1})], \tag{A14}$$

$$\phi_{jt} + \mu^B_{jt} = \lambda_{jt}\mathbb{E}[\frac{S_{jt+1}}{S_{jt}}(1 + i^*_t)], \tag{A15}$$

$$\phi_{jt} = \lambda_{jt} + \delta_{jt}n_{jt}, \tag{A16}$$

$$\delta_{jt}M_{jt} = \lambda_{jt}\gamma_{jt}W_{jt}, \tag{A17}$$

where δ_{jt} , ϕ_{jt}, and λ_{jt} are multipliers on constraints ($A1$), ($A4$), and ($A5$). μ^B_{jt} and μ^b_{jt} are multipliers on the trading constraints of the nominal and real bonds, respectively.

Deriving the Fisher equation and the parity conditions As in the problem of country 1, we combine ($A12$) and ($A13$) and get

$$\mathbb{E}[\frac{P_{jt+1}}{P_{jt}}(1 + r_{jt+1})] = (1 + i_{jt}),$$

which is the Fisher equation for this country.

Combining equations ($A12$) and ($A14$), we can get

$$\lambda_{jt}\mathbb{E}[\frac{S_{jt+1}P^*_{t+1}}{S_{jt}P^*_t}(1 + r^*_{t+1})] - \mu^d_{jt} = \lambda_{jt}\mathbb{E}[\frac{P_{jt+1}}{P_{jt}}(1 + r_{jt+1})]$$

or

$$\mathbb{E}\{[\frac{\frac{S_{jt+1}P^*_{t+1}}{P_{jt+1}}}{\frac{S_{jt}P^*_t}{P_{jt}}}(1 + r^*_{t+1})] - \frac{P_{jt}}{P_{jt+1}}\frac{\mu^d_{jt}}{\lambda_{jt}} - (1 + r_{jt+1})\} = 0$$

i.e.

$$\mathbb{E}\{[\frac{\xi_{jt+1}}{\xi_{jt}}(1 + r^*_{t+1})] - \frac{P_{jt}}{P_{jt+1}}\frac{\mu^d_{jt}}{\lambda_{jt}}\} = (1 + r_{jt+1}).$$

So, we can write this in the Fisher equation and obtain

$$\mathbb{E}[\frac{P_{jt+1}}{P_{jt}}\frac{\xi_{jt+1}}{\xi_{jt}}(1 + r^*_{t+1})] - \frac{\mu^d_{jt}}{\lambda_{jt}} = (1 + i_{jt}).$$

to the extent that the growth rate of the real exchange rate and the deviations of UIP are small at the frequency we are interested (over 4 years) we can approximate the behavior of the real rate in country j using the real rate in the USA.

B Data

B.1 The United States

The series of nominal GDP, the three-month Treasury bill rate, currency in circulation, and "standard" M1 are collected from FRED.[2] Currency and the three-month T-bill rate are used as the measures of cash and the interest rate associated with it.

NewM1 The construction of NewM1 follows Lucas and Nicolini (2015):

$$\text{NewM1} = \text{M1} + \text{MMDAs}.$$

The Money Market Demand Accounts (MMDAs) series are constructed by aggregating term RCON6810 under Schedule RC-E from individual banks' call reports. The original data are publicly available at the Central Data Repository Public Data Distribution website of Federal Financial Institutions Examination Council.[3]

The MMDAs series have been issued since 1982Q3, but the data are available only after 1984Q2. We apply a linear interpolation of money growth rates for the periods in between. Figure A1 depicts the money growth rates of cash, the "standard" M1, and the New M1 series since 1960.

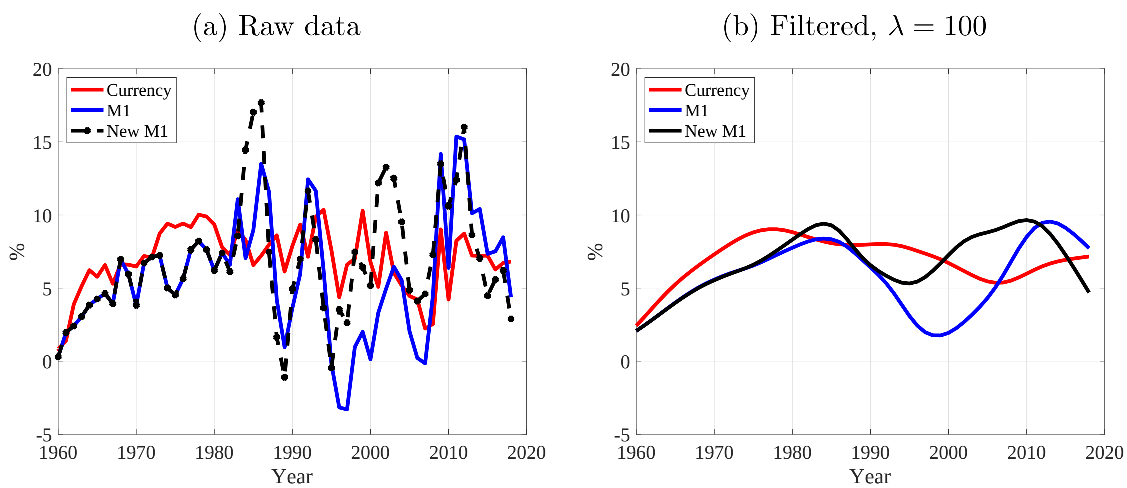


Figure A1: Money Growth in the United States

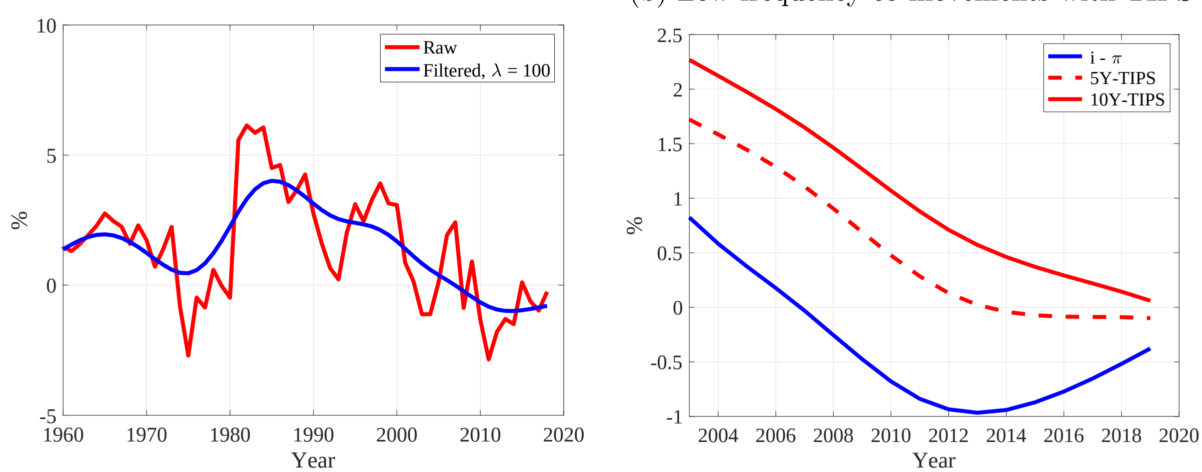
Imputed interest rate We impute the interest rate associated with the New M1 by subtracting the fraction of interest paid by deposits and by MMDAs from the three-month T-bill rate; that is,

$$\tilde{r} = r^{3\text{m}} - s_d i^d - s_a i^a,$$

[2] FRED: https://fred.stlouisfed.org/.
[3] FFIEC: https://cdr.ffiec.gov/public/.

where s_d and s_a are the ratio of deposits to NewM1 and the ratio of MMDAs to NewM1, and $i^{3\text{m}}$,i^d, and i^a are the interest rates on three-month T-bills, deposits, and MMDAs, respectively.

Real interest rates The real interest rate is constructed by subtracting the three-month T-bill rate by inflation. In view of the lack of real interest rates for other countries, we use the real rates of the United States as the proxy of real rates in other countries for the quantitative illustration of Fisher equation. Figure A2(a) plots the constructed raw series of US real interest rates since 1960 and the HP-filtered series using smoothing parameter 100. Figure A2(b) compares the imputed real interest rates with interest rates on Treasury Inflation-Indexed Securities (TIPS) at five- and ten-year maturities. As can be seen from Figure A2(b), the difference between our imputed real interest rates and interest rates on long-term TIPS is very stable over time.



(a) Imputed real interest rates

(b) Low-frequency co-movements with TIPS

Figure A2: Imputation of US Real Interest Rates

B.2 Other OECD countries

We need data for prices, money stock M1, GDP, and interest rates for each country. In view of the lack of real GDP, we collect data of nominal GDP in local currency and impute real GDP with prices. The main source for nominal interest rates and M1 is the OECD data website, and the main source for nominal GDP is the International Financial Statistics (IFS) of the International Monetary Fund (IMF).[4] We collect data starting from 1960 for all countries as long as there is availability. For countries with missing values up till 1960, we splice the series from the OECD and the IFS with data constructed in Benati et al. (2021).[5] Money data for countries in the eurozone (Germany, Italy, Netherlands, Portugal, and Spain) are available only up till 1998.

We finally have data for 16 OECD countries other than the United States and break them into two groups based on the similarity of inflation movements:

1. USA, Australia, Canada, Denmark, Germany, Japan, New Zealand, and the UK;
2. Italy, Netherlands, Portugal, South Korea, Spain, Colombia, Chile, Mexico, and Turkey.

The following list details special issues in the construction of the dataset.

Australia Interest rates in 1960–1967 and M1 in 1960 are spliced with Benati et al. (2021).

Canada Nominal GDP in 1960 is spliced using Benati et al. (2021). Between 1982 and 2005, M1 in the OECD dataset has faster growth at the beginning and lower growth in later years than the M1 data in Benati et al. (2021), which results in a similar cumulative growth across these two sources.

Chile We use data for Chile after 1985, because in the 1970s, Chile had several years of hyperinflation over 100%. Interest rates in 1985–1997 are spliced with Benati et al. (2021).

Colombia The OECD provides nominal interest rates only after 1986. Interest rates in Benati et al. (2021) and the OECD behave similarly after 1995 but are significantly higher in the OECD than in Benati et al. (2021). For consistency, we use Benati et al. (2021) for interest rates in all periods .

Denmark Interest rates between 1960 and 1986 are spliced using Benati et al. (2021).

Germany The IFS provides nominal GDP only after 1992. For consistency, we use Benati et al. (2021) for nominal GDP in all periods.

[4]OECD data: https://data.oecd.org/; IFS data: https://data.imf.org/.

[5]See Benati et al. (2019) for more details about the original data sources.

Italy Interest rates before 1979 are spliced using Benati et al. (2021). The IFS provide nominal GDP only after 1995, and the OECD does not have data for M1. We use Benati et al. (2021) for nominal GDP and M1 in all periods.

Japan Interest rates before 2003 are spliced using Benati et al. (2021).

Mexico Interest rates before 1997, prices before 1969, and M1 before 1977 are spliced using Benati et al. (2021). Nominal GDP for all years is taken from Benati et al. (2021).

Netherlands Interest rates before 1982 and nominal GDP before 1995 are spliced using Benati et al. (2021). M1 for all years is taken from Benati et al. (2021).

New Zealand Interest rates before 1974, nominal GDP before 1970, and M1 before 1978 are spliced using Benati et al. (2021).

Portugal Interest rates before 1986 and nominal GDP before 1995 are spliced using Benati et al. (2021). M1 for all years is taken from Benati et al. (2021).

South Korea Interest rates are taken from Benati et al. (2021).

Spain Interest rates before 1976 are spliced using Benati et al. (2021). Note that interest rates between 1977 and 1981 in the OECD dataset are higher than those in Benati et al. (2021). The IFS provide nominal GDP since 1995. We use Benati et al. (2021) for nominal GDP in all periods for consistency.

Turkey Data for Turkey are available from 1969 onwards. Nominal GDP before 1987 is spliced using Benati et al. (2021). Interest rates for all years are taken from Benati et al. (2021).

The UK We use all variables for all years from Benati et al. (2021).

Table A1 provides the summary statistics of mean and standard deviation of inflation π, nominal interest rate i, money growth μ, and real GDP growth g by country.

Table A1: Summary Statistics of Main Variables

Country	Periods	π	i	μ	g
USA - Currency	1960–2005	4.26	6.16	7.25	2.94
		(2.91)	(3.13)	(2.34)	(2.43)
USA - Standard M1	1960–2005	4.26	6.16	5.14	2.94
		(2.91)	(3.13)	(3.71)	(2.43)
USA - New M1 - Interp1	1960–2005	4.26	4.97	7.32	2.94
		(2.91)	(2.57)	(6.16)	(2.43)
USA - New M1 - Interp2	1960–2005	4.26	4.97	6.50	2.94
		(2.91)	(2.57)	(4.11)	(2.43)
Australia	1960–2005	5.48	8.28	9.01	3.73
		(4.03)	(4.06)	(6.21)	(2.74)
Canada	1960–2005	4.36	7.18	8.06	3.70
		(3.18)	(3.50)	(4.58)	(2.95)
Denmark	1960–2005	5.45	9.89	10.79	2.73
		(3.55)	(4.39)	(6.26)	(2.39)
Germany	1961–2005	3.00	5.61	8.18	3.16
		(1.80)	(2.53)	(3.51)	(2.95)
Japan	1960–2005	3.85	4.22	11.36	4.51
		(4.37)	(2.60)	(7.07)	(4.94)
New Zealand	1960–2005	6.56	9.65	8.82	2.88
		(5.38)	(4.50)	(7.62)	(2.35)
The UK	1960–2005	6.38	8.35	9.72	2.75
		(5.44)	(3.57)	(6.13)	(2.04)
Italy	1960–2005	7.14	6.56	13.14	4.26
		(5.63)	(3.64)	(6.58)	(2.79)
Netherlands	1962–2005	3.96	5.29	7.72	3.42
		(2.59)	(1.96)	(4.98)	(2.88)
Portugal	1960–2005	10.28	8.35	12.99	4.87
		(8.13)	(6.62)	(6.64)	(3.32)
South Korea	1962–2005	9.73	10.06	24.47	9.88
		(7.57)	(7.18)	(12.39)	(6.21)
Spain	1960–2005	7.92	8.85	12.99	4.87
		(5.57)	(5.08)	(6.64)	(3.32)
Chile	1980–2005	13.39	26.24	22.01	5.03
		(9.70)	(18.00)	(14.04)	(5.84)
Colombia	1960–2005	17.59	9.20	21.70	4.15
		(7.65)	(6.11)	(7.66)	(2.15)
Mexico	1960–2005	24.43	24.32	28.22	5.02
		(31.02)	(21.68)	(24.00)	(5.10)
Turkey	1969–2005	46.80	36.52	48.36	4.75
		(27.75)	(20.98)	(22.46)	(7.14)

B.3 Additional filtering results

HP filter with $\lambda = 6.5$ In Figure A5 and Figure A6, we report the two illustrations when we filter series using smoothing parameter $\lambda = 6.5$. Apparently, as $\lambda = 6.5$ does not remove the "tightening cycles", the fit of theory is worsen compared when we use $\lambda = 100$.

BP filter We also consider band pass filter proposed by Christiano and Fitzgerald (2003). As shown in Figure A3, the BP filter that filters components higher than frequency of 24 years removes most of the "tightening cycles". Figure A4 further shows that it achieves similar performance than using HP filter with smoothing parameter $\lambda = 100$.

In practice, we find that the BP filter renders similar results except for it does a relatively bad job in handling multiple regimes, such as periods of interest rate repression at the beginning of observations in a few countries, like Italy and Portugal. In Figure A7 and Figure A8, we report the two illustrations with the BP filter.

(a) Low frequency component (b) High frequency component

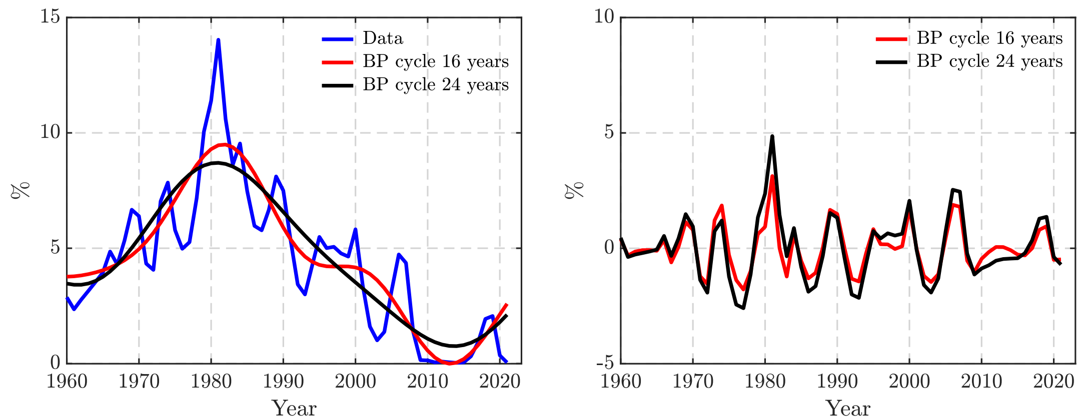


Figure A3: BP Filter by Frequency

(a) Low frequency component (b) High frequency component

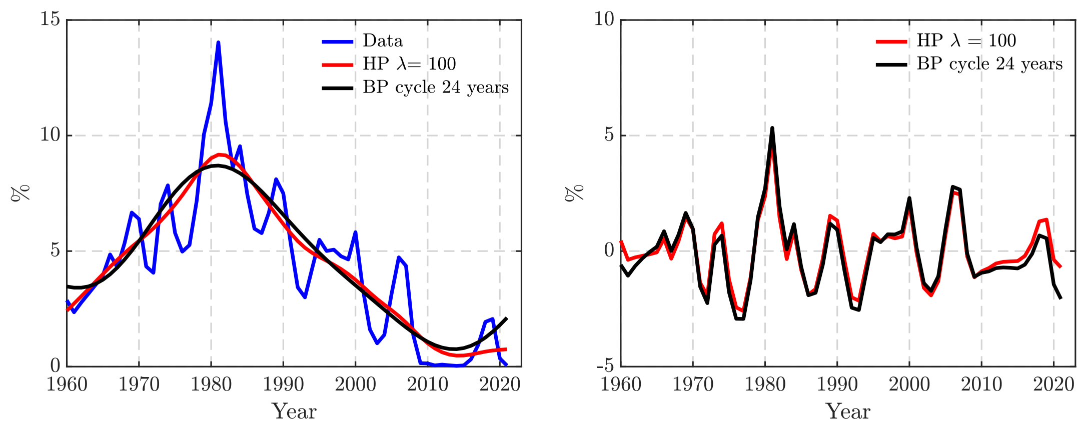


Figure A4: Comparison between the HP and BP Filter

Illustration 1

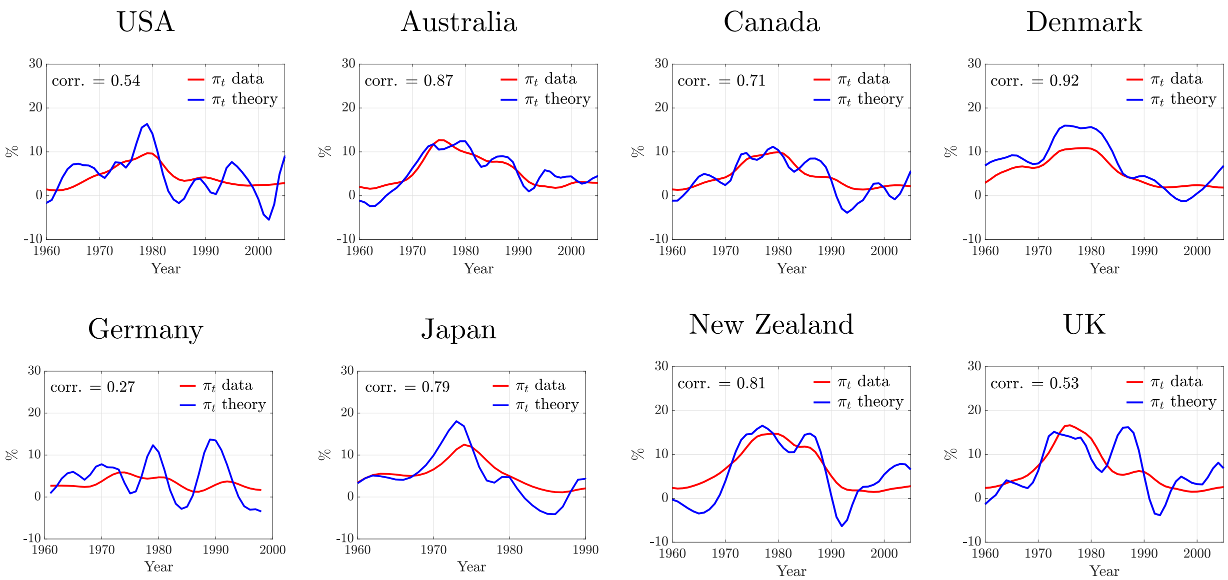


Illustration 2

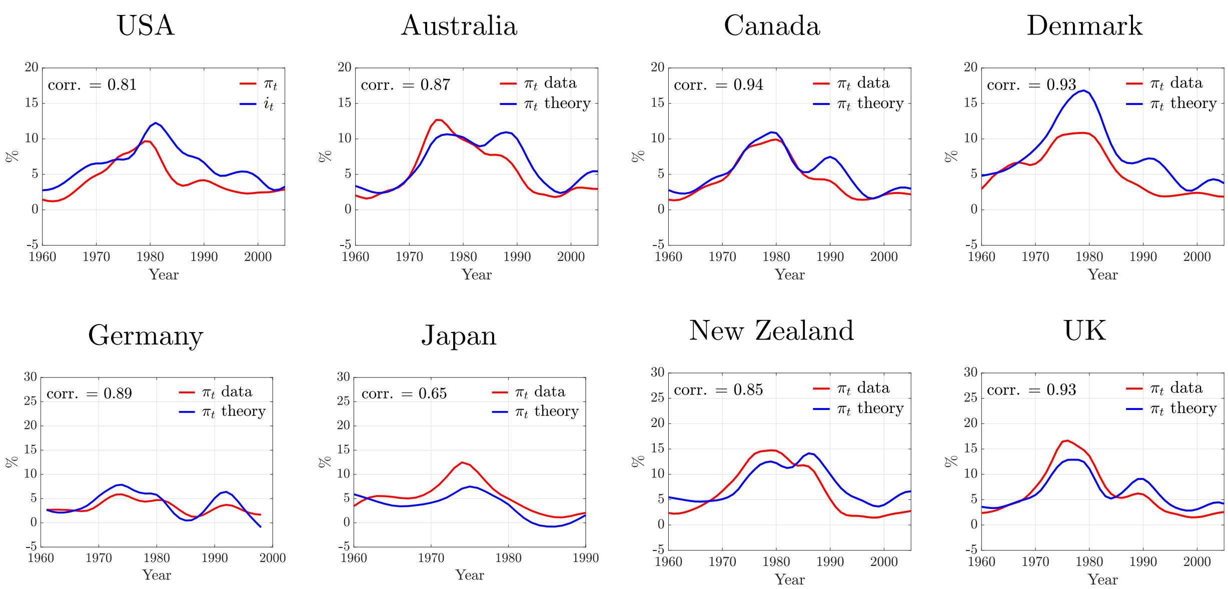


Figure A5: Countries in Group 1, HP Filter, $\lambda = 6.5$

Illustration 1

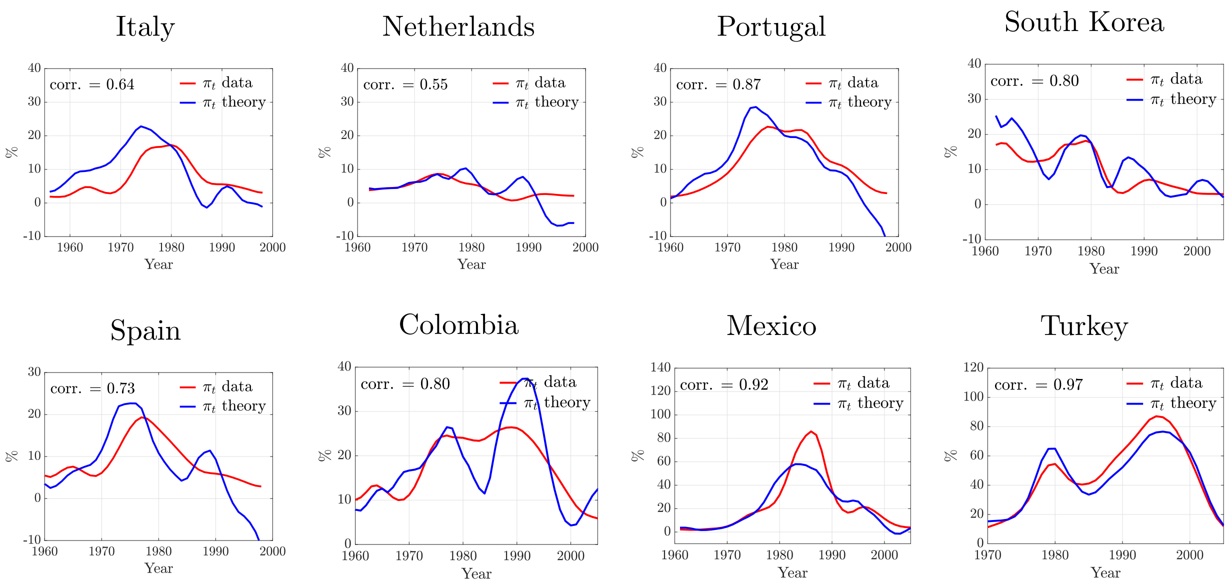


Illustration 2

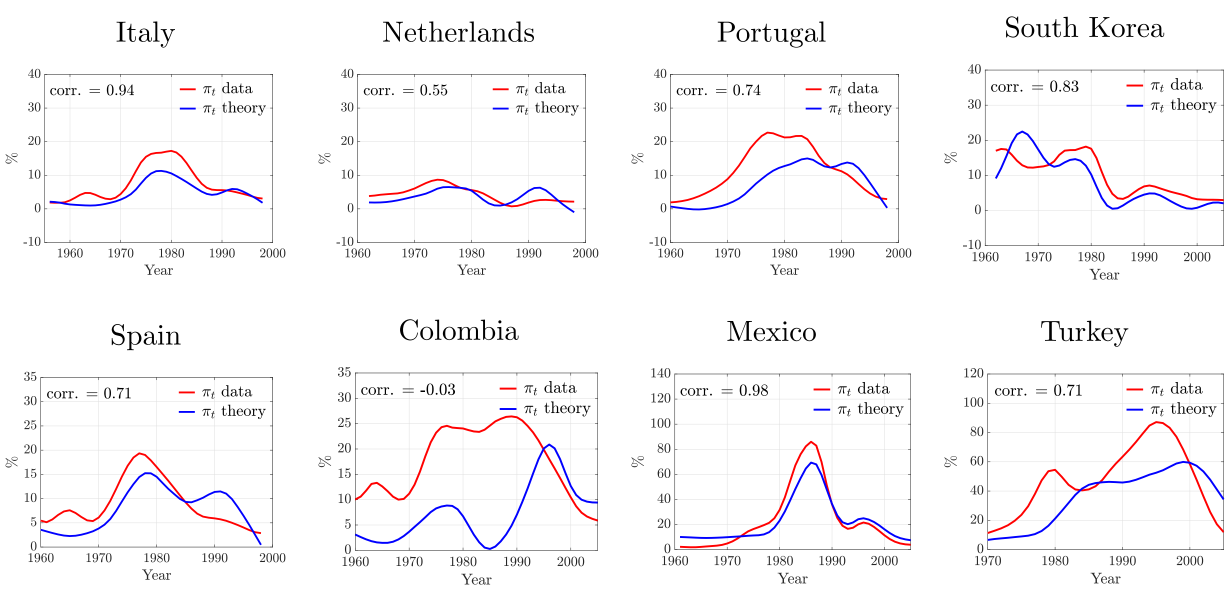


Figure A6: Countries in Group 2, HP Filter, $\lambda = 6.5$

Illustration 1

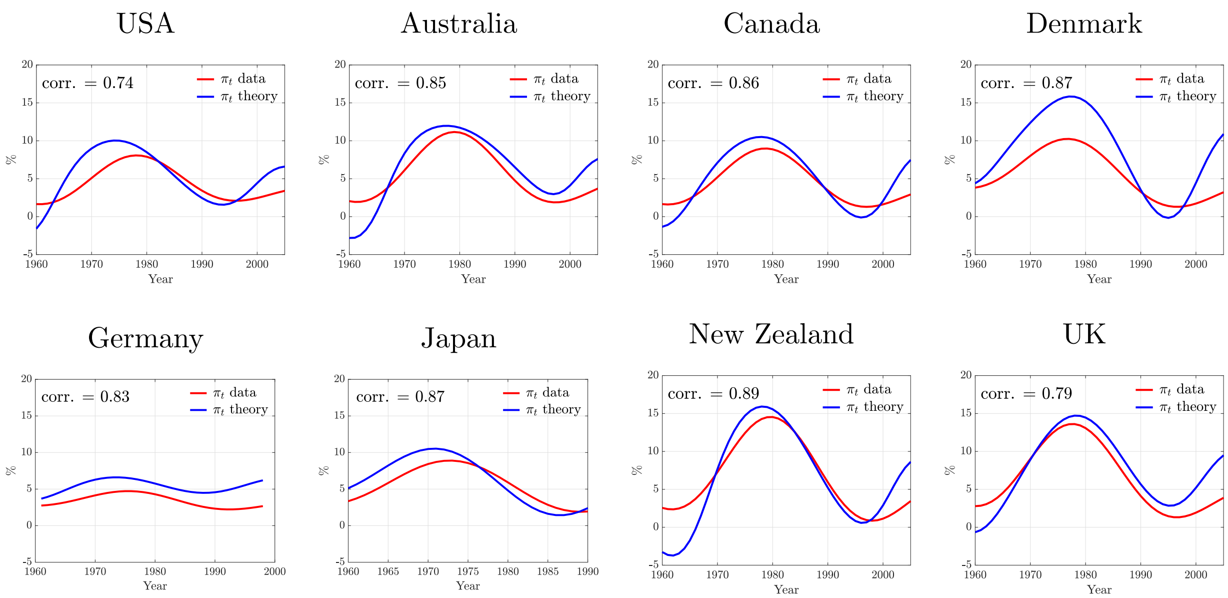


Illustration 2

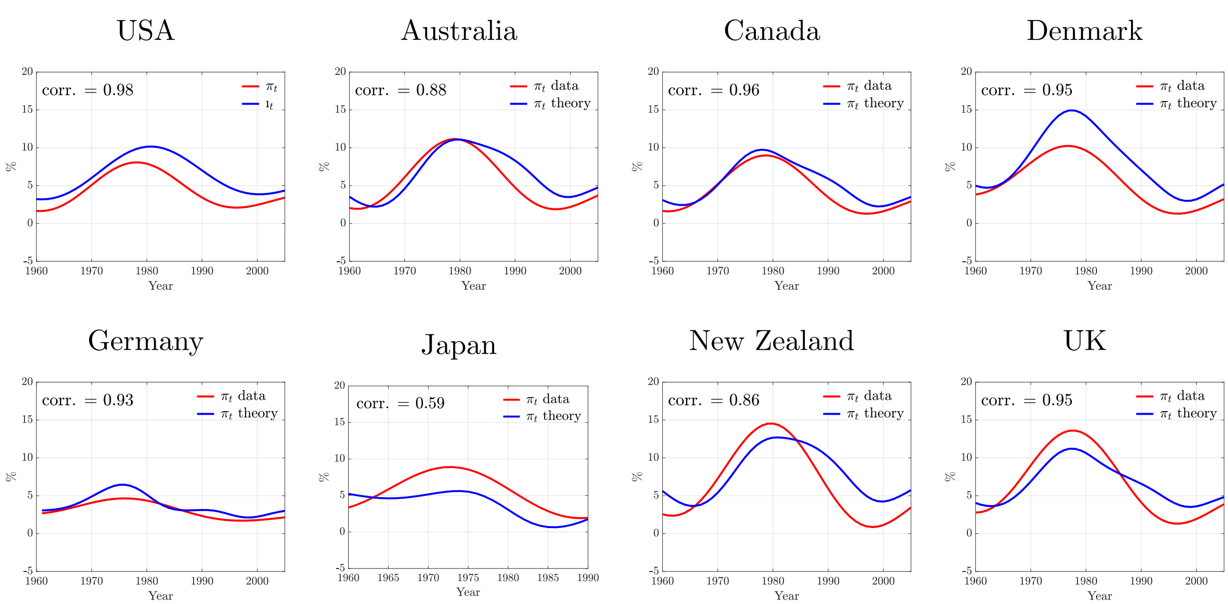


Figure A7: Countries in Group 1, BP Filter

Illustration 1

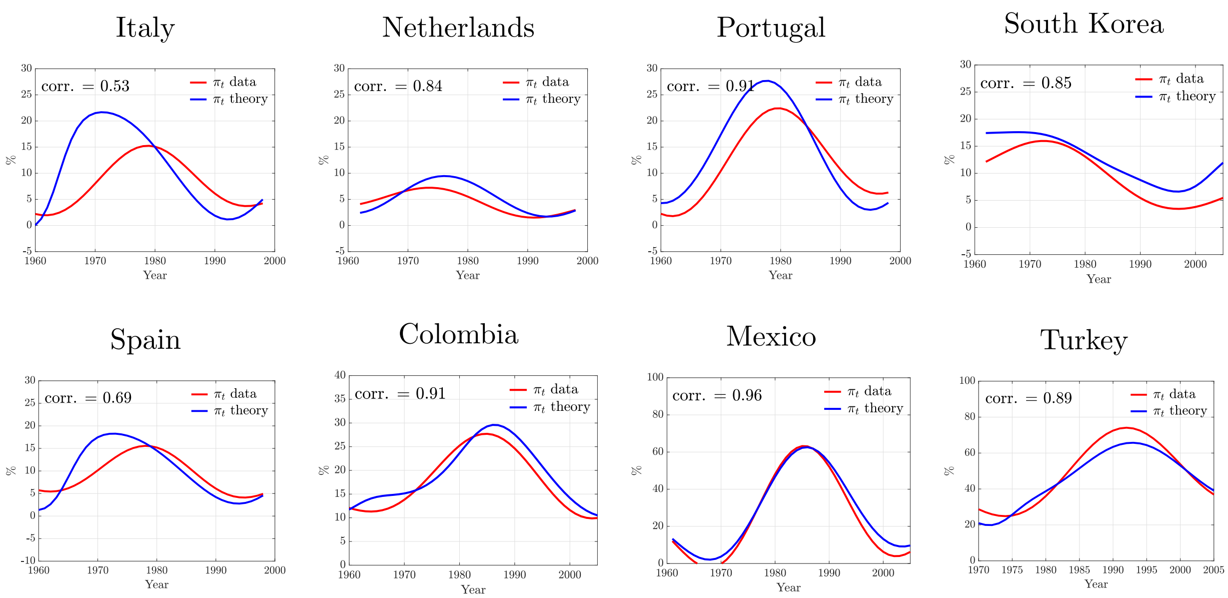


Illustration 2

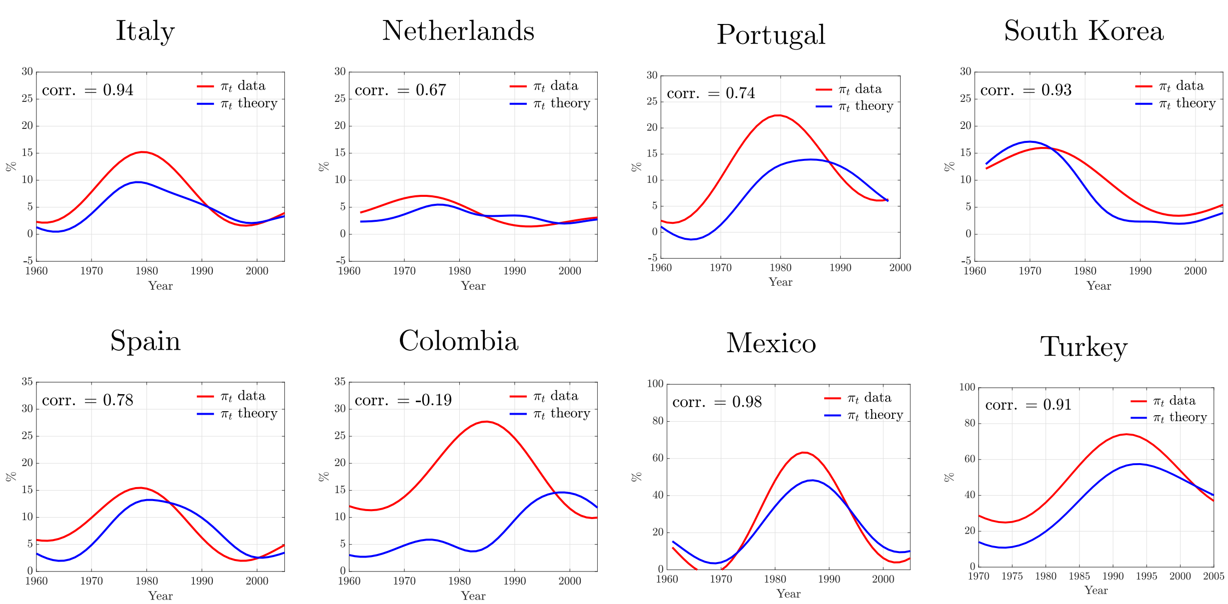


Figure A8: Countries in Group 2, BP Filter

C Estimation Details of the Baseline Model

The model of Section 4 is estimated using Bayesian methods. We jointly estimate the structural parameters, ϑ, and the dates of regime changes, $\mathbf{T}$. We estimate, T^{on}, T^{off} and T_κ. The first two correspond to the dates of the high inflation regime for which $\mathbb{I}^s = 1$. In the case of T^{off}, we sample from a uniform distribution over 1979q4 and 1983q4, which corresponds to the Volcker disinflation; T_κ is the date break for the variance of all structural shocks except for that of the money demand shock. The variance of the remaining structural shocks shifts proportionally at T_κ by a factor of κ, so that the variance covariance matrix shifts from $\kappa\Omega$ to Ω. This specification serves two purposes: First, it helps the model capture the decrease in volatility associated with the Great Moderation. Second, and more important for our purposes, is that it guards against the possibility that the estimation relies on shocks to the inflation target to account for the increased volatility of the 1970s. For the variance of shocks to money demand, σ_ξ, the volatility shifts in 1982q4 to $\kappa_m\sigma_\xi$, which, as explained above, lines up with the regime change in the measurement of M1 explained in Lucas and Nicolini (2015).

The model is estimated on real GDP per capita growth; the federal funds rate; core inflation as measured by the CPI, excluding food and energy, the Michigan survey measure of inflation expectations; and money growth.

To construct the likelihood of the model under regime changes, we use the method outlined in Kulish and Pagan (2017). That method deals with a more general case than the application we are considering, so we provide a brief discussion of the case we deal with here.

Let $t = 1, 2, \ldots, T$ index the observations in the sample. From period $t = 1, 2, \ldots, T^{\text{on}}-1$, the steady state level of inflation is π. The first-order approximation to the equilibrium conditions around this initial steady state is given by the linear rational expectations system of n equations that we write as

$$A_0\mathbf{y}_t = C_0 + A_1\mathbf{y}_{t-1} + B_0\mathbb{E}_t\mathbf{y}_{t+1} + D_0\varepsilon_t, \tag{A18}$$

where A_0, C_0, A_1, B_0 and D_0 are the structural matrices of the initial steady-state, $\mathbf{y}_t$ is a $n \times 1$ vector of state and jump variables, and ε_t is an $l \times 1$ vector of exogenous $i.i.d$ shocks. The unique rational expectations solution to (A18) is

$$\mathbf{y}_t = C + Q\mathbf{y}_{t-1} + G\varepsilon_t. \tag{A19}$$

For $t = T^{\text{on}}$ until $T^{\text{off}} - 1$ the steady state level of inflation increases to $\pi + \Delta\pi$ and $\mathbb{I} = 1$, so the structural equations are given by

$$\bar{A}_0\mathbf{y}_t = \bar{C}_0 + \bar{A}_1\mathbf{y}_{t-1} + \bar{B}_0\mathbb{E}_t\mathbf{y}_{t+1} + \bar{D}_0\varepsilon_t, \tag{A20}$$

with solution

$$\mathbf{y}_t = \bar{C} + \bar{Q}\mathbf{y}_{t-1} + \bar{G}\varepsilon_t. \tag{A21}$$

At T^{off} the economy reverts to (A18) with steady state π. These structural changes imply that the reduced form is time-varying over the sample. In general,

$$\mathbf{y}_t = C_t + Q_t \mathbf{y}_{t-1} + G_t \varepsilon_t. \tag{A22}$$

With a sample of data, $\{y_t^{obs}\}_{t=1}^{T}$, where y_t^{obs} is a $n_{obs} \times 1$ vector of observable variables that relates to the model's variables through the measurement equation below:

$$y_t^{obs} = H_t \mathbf{y}_t. \tag{A23}$$

Here, H_t is time varying to account for the fact that the Michigan measure of inflation expectations becomes available only after 1978. The observation equation, Equation (A23), and the state equation, Equation (A22), form a state-space model. The Kalman filter can be used to construct the likelihood function for the sample $\{y_t^{obs}\}_{t=1}^{T}$, given by $\mathcal{L}(Y|\vartheta, \mathbf{T})$ as outlined in Kulish and Pagan (2017).

Given the joint posterior of the structural parameters and the date breaks, $p(\vartheta, \mathbf{T}|Y) = \mathcal{L}(Y|\vartheta, \mathbf{T})p(\vartheta)p(\mathbf{T})$, we simulate from this distribution using the Metropolis-Hastings algorithm as used by Kulish and Rees (2017). As we have continuous and discrete parameters, we separate them into two blocks: one for date breaks and one for structural parameters. The sampler delivers draws from the joint posterior of both sets of parameters.

Below we report results from our baseline estimation. We also estimated the model using cash, rather than M1. We also estimated the slope of the Phillips curve rather than calibrating it. None of this variations altered the main results reported in this section.

C.1 Long-run inflation expectation

Our model is also consistent with the evidence in Hazell, Herreño, Nakamura, and Steinsson (2022), in that the main driver of the long-run component of inflation in the USA since the 60s are unrelated to Phillips curve considerations. As we show below, our inflation target shock strongly co-moves with long-run inflation expectations, the main driver of inflation in their model.

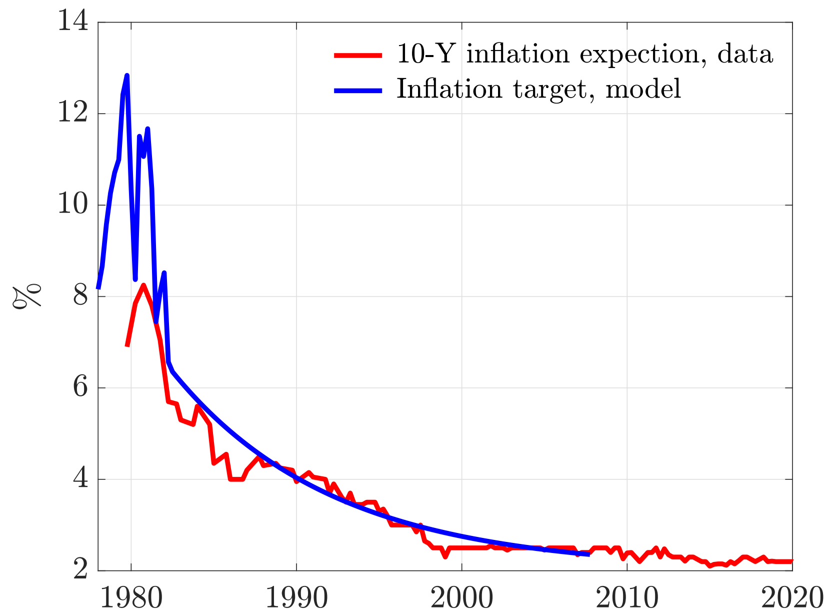


Figure A9: Long-run Inflation Expectation and Inflation Target

Notes: The 10-year ahead inflation expectation is constructed by combining the Survey of Professional Forecasters for the years after 1990 and the Blue Chip available on the Data site of Federal Reserve Bank of Philadelphia for the 1980s, following Hazell et al. (2022).

C.2 Prior and posteriors of the structural parameters

Table A2: Estimation Results – Interest Rate Rule

	Prior distribution			Posterior distribution			
	Shape	Mean	Std Dev.	Mode	Mean	5 %	95 %
Standard Deviations							
$100 \times \sigma_i$	Inv. Gamma	1	2	0.08	0.08	0.06	0.10
$100 \times \sigma_a$	Inv. Gamma	1	2	1.45	1.33	1.17	1.84
$100 \times \sigma_e$	Inv. Gamma	1	2	0.11	0.11	0.09	0.14
$100 \times \sigma_z$	Inv. Gamma	1	2	0.47	0.47	0.42	0.53
$100 \times \sigma_\pi$	Inv. Gamma	1	2	0.10	0.10	0.09	0.12
$100 \times \sigma_\tau^{obs}$	Inv. Gamma	1	2	0.16	0.15	0.12	0.20
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.25	1.25	1.11	1.41
κ	Normal	2	0.3	2.00	1.99	1.82	2.20
κ_m	Normal	2	0.3	1.37	1.32	1.14	1.61
Structural parameters							
ρ_i	Beta	0.5	0.2	0.90	0.90	0.86	0.93
ϕ_π	Normal	2	0.5	2.06	2.12	1.40	2.74
ϕ_x	Normal	0.125	0.05	0.37	0.37	0.31	0.42
$10 \times \omega$	Normal	0.5	0.1	0.52	0.53	0.37	0.69
η	Normal	0.5	0.05	0.46	0.46	0.38	0.55
ρ_m	Beta	0.5	0.2	0.95	0.96	0.91	0.98
ρ_a	Beta	0.5	0.2	0.88	0.88	0.84	0.91
ρ_e	Beta	0.5	0.2	0.46	0.47	0.34	0.56
ρ_τ	Beta	0.5	0.2	0.79	0.80	0.68	0.89
ρ_π	Beta	0.5	0.2	0.97	0.98	0.97	0.98
$100 \times \Delta_\pi$	Uniform	[-2 , 6]		1.03	0.98	0.18	1.95
ρ_ξ	Beta	0.5	0.2	0.60	0.59	0.46	0.74

(a) T^{on} (b) T^{off} (c) T_κ

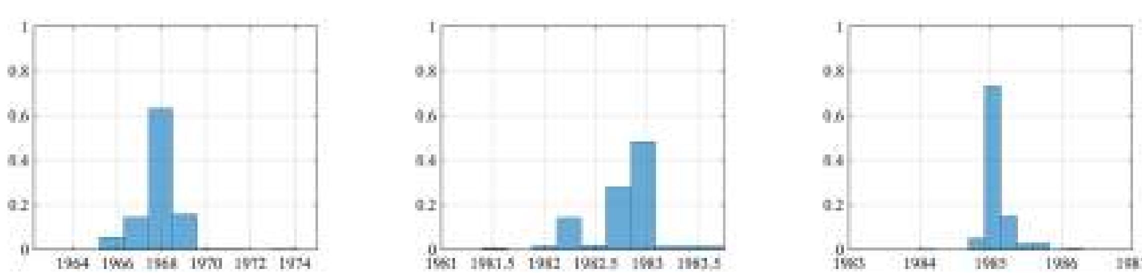


Figure A11: Posterior Distributions of Regime Switching Dates – Interest Rate Rule

C.3 Additional model fitness

In Section 4.3 of our main text, we evaluate the sensitivity of filter to price-setting frictions and report the results for inflation. In Figure A12, we further report the results for the interest rate.

Recall that also in Section 4.3, we fix model parameters to mode estimates, and simulate the model by feeding in the estimated inflation target shocks and drawing all other shocks randomly. Figure A13 reports further the fitness of the interest rate and money growth.

In Figure A14, we further sample parameters using the posterior distribution and repeat the exercises. We consider two ways of sampling the parameters. On the top panel of Figure A14, we report results when we sample all model parameters using the posterior. On the bottom, we sample only parameters that are associated with the inflation target process.

(a) Simulation with regime changes (b) Simulation without regime changes

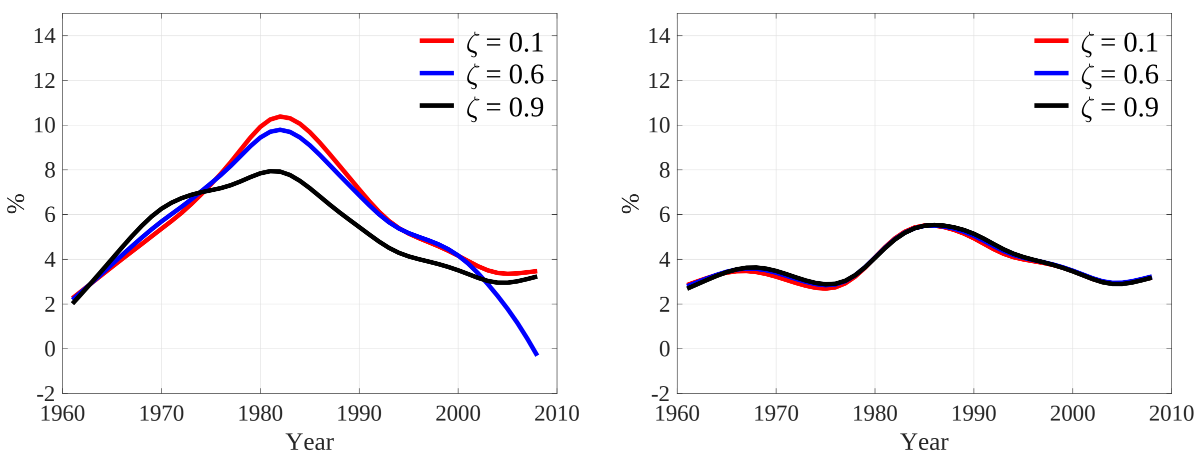
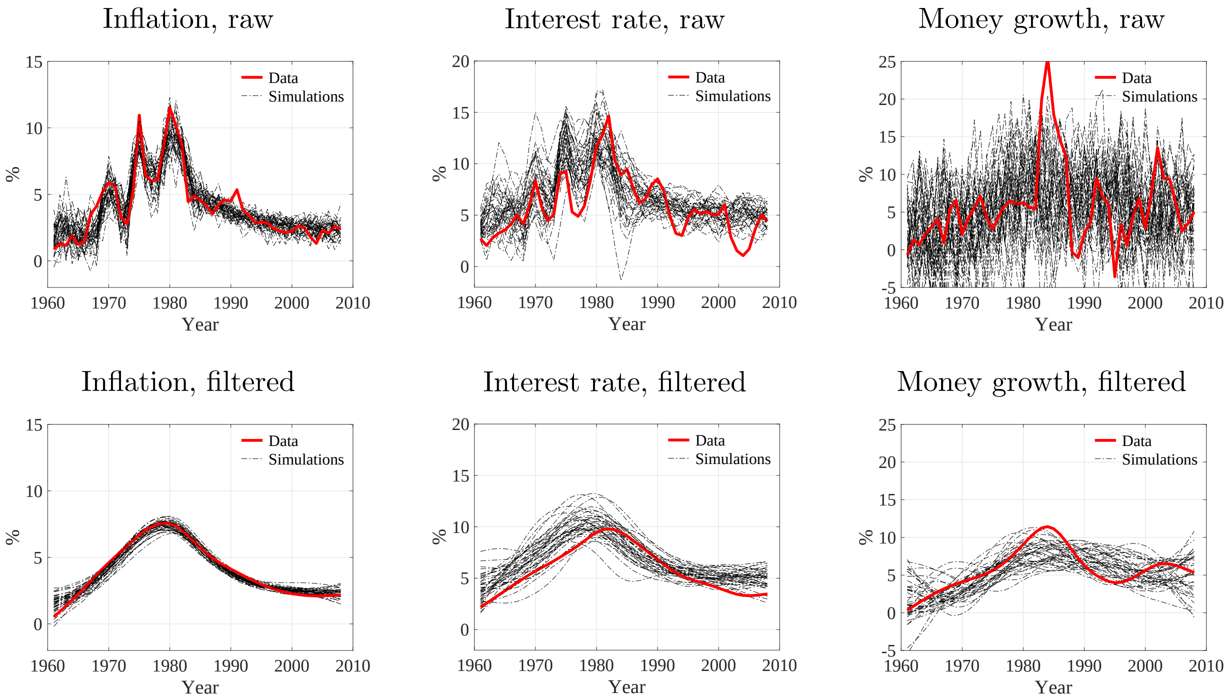


Figure A12: HP-filtered Interest Rates by Degree of Price Stickiness – Interest Rate Rule

Estimation with regime changes



Estimation without regime changes

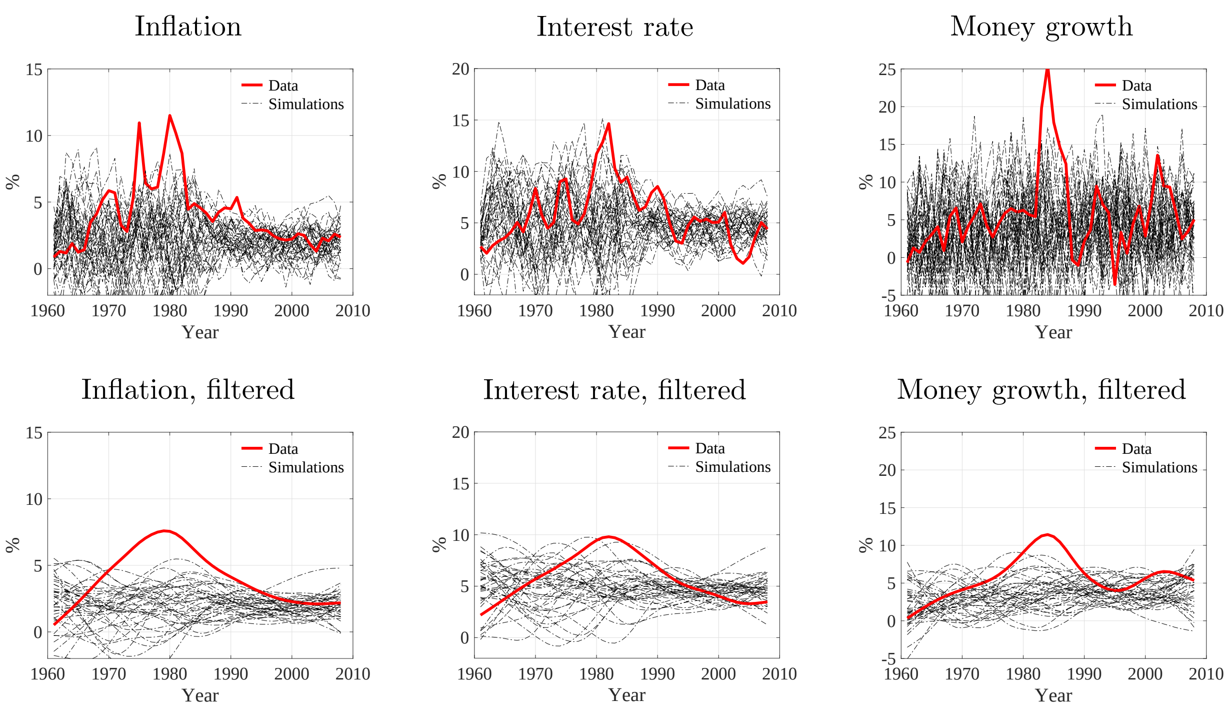


Figure A13: Model Fitness – Interest Rate Rule

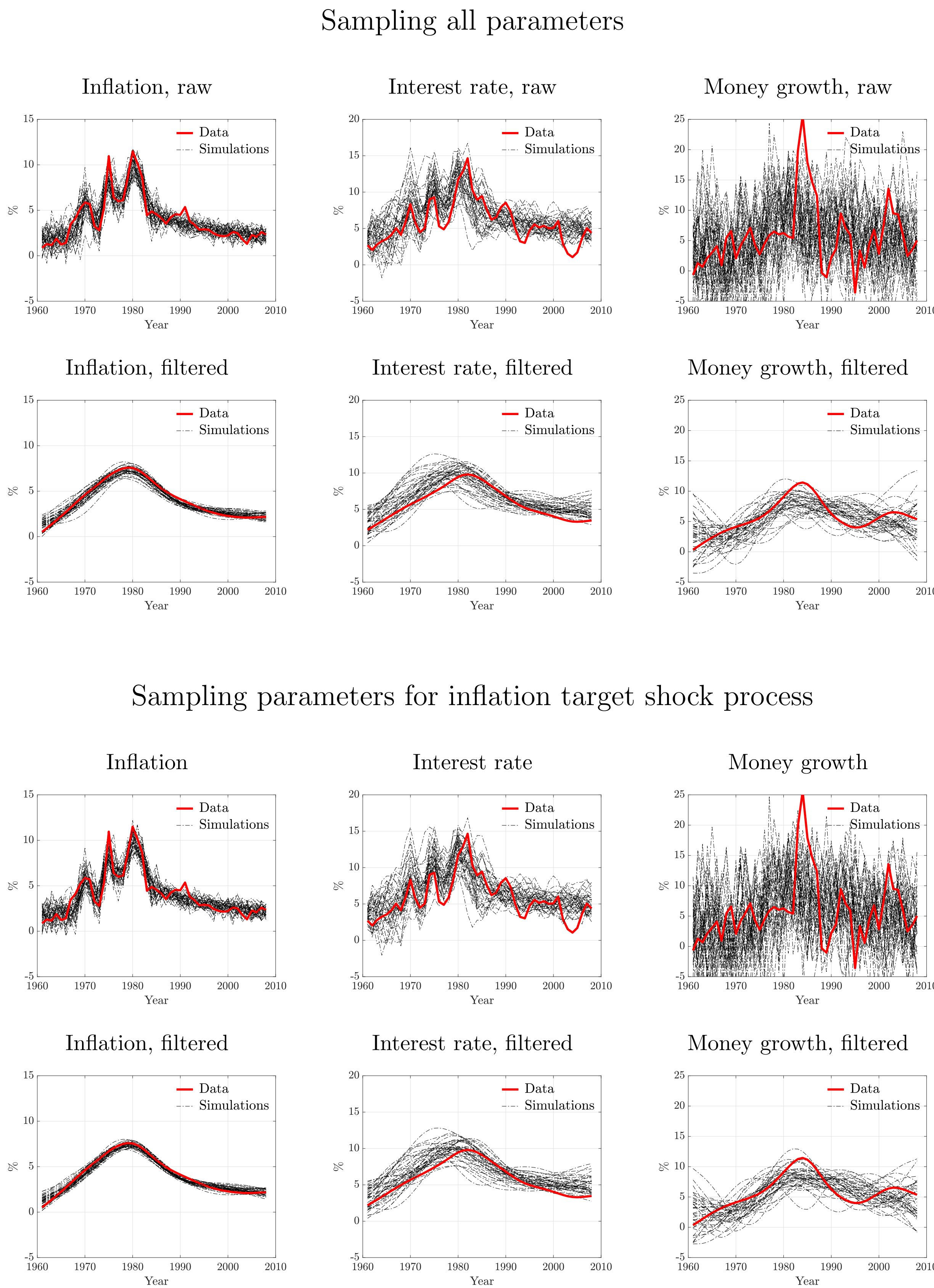


Figure A14: Model Fitness – Interest Rate Rule

D Alternative Models

D.1 Model with a money rule

Model setup Note that the model with interest rate rule is block recursive in the sense that conditional on the shocks, inflation, interest rate, and output gap are determined by equations (7), (8), and (9). Money is thus determined by equation (17) outside the equations block.

Under a money rule, the model is not block recursive anymore. But we can combine the Euler equation (7) with the money demand equation (17) to eliminate the interest rate. The rest of the system can then be used to solve for inflation, money growth and the output gap. As we did when we considered interest rate rules, we allow for regime changes in the target for money growth. Thus, μ_t^* evolves according to

$$\mu_t = \mu_t^* + \rho_\mu \left(\mu_{t-1} - \mu_{t-1}^*\right) + \theta_\pi \left(\pi_t - \pi_t^*\right) + \theta_x x_t + \varepsilon_{\mu,t} \tag{A24}$$

$$\mu_t^* = (1-\rho^*)\mu^s + \rho^* \mu_{t-1}^* + \mathbb{I}^s \varepsilon_{\mu^*,t}, \tag{A25}$$

where ρ_μ is the response to deviation of money growth, and θ_π and θ_x capture the responses of policy to inflation deviations and output gaps. To solve the model, we use equation (A25) as a replacement of equation (10). The regime indicator function $\mathbb{I}^s$ is defined in same way as in equation (12), and the implied target inflation is given by

$$\pi_t^* = \mu_t^* - z. \tag{A26}$$

Estimation results We estimate the model using the same observable variables that we used for the case of interest rate rule. We treat money stock between 1980Q1 and 1984Q2 as unobservables. This quantitatively makes no impact on our estimation of the structural parameters. Table A3 reports the estimated parameters and Figure A15 reports distributions of regime switching dates. Our estimation again detects that the monetary policy regime switches between the mid 1960s and early 1980s.

Table A3: Estimation Results – Model with Money Rule

	Prior distribution			Posterior distribution			
	Shape	Mean	Std Dev.	Mode	Mean	5 %	95 %
Standard Deviations							
$100 \times \sigma_{mu}$	Inv. Gamma	1	2	0.90	0.86	0.77	1.05
$100 \times \sigma_a$	Inv. Gamma	2	3	2.60	2.57	1.17	1.84
$100 \times \sigma_e$	Inv. Gamma	1	2	0.10	0.09	0.08	0.13
$100 \times \sigma_z$	Inv. Gamma	1	2	0.50	0.50	0.424	0.57
$100 \times \sigma_{\mu^\star}$	Inv. Gamma	1	2	0.10	0.10	0.09	0.12
$100 \times \sigma_\tau$	Inv. Gamma	1	2	0.13	0.14	0.10	0.17
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.36	1.35	1.25	1.48
κ	Normal	2	0.3	2.02	2.00	1.75	2.21
Structural parameters							
ρ_μ	Beta	0.5	0.2	0.30	0.27	0.21	0.38
θ_π	Normal	1	0.2	1.25	1.23	0.90	1.60
θ_x	Normal	4	0.5	4.16	4.13	3.44	4.80
$10 \times \omega$	Normal	0.5	0.1	0.49	0.42	0.29	0.68
η	Normal	0.15	0.025	0.13	0.14	0.11	0.15
ρ_m	Beta	0.5	0.2	0.19	0.19	0.12	0.28
ρ_a	Beta	0.5	0.2	0.94	0.94	0.91	0.96
ρ_e	Beta	0.5	0.2	0.45	0.39	0.35	0.56
ρ_τ	Beta	0.5	0.2	0.80	0.81	0.73	0.88
ρ_π	Beta	0.5	0.2	0.98	0.98	0.97	0.98
$100 \times \Delta_\pi$	Uniform	[-2 , 6]		1.83	2.00	1.37	2.23
ρ_ξ	Beta	0.5	0.2	0.99	0.99	0.99	1.00

(a) T^{on} (b) T^{off} (c) T_κ

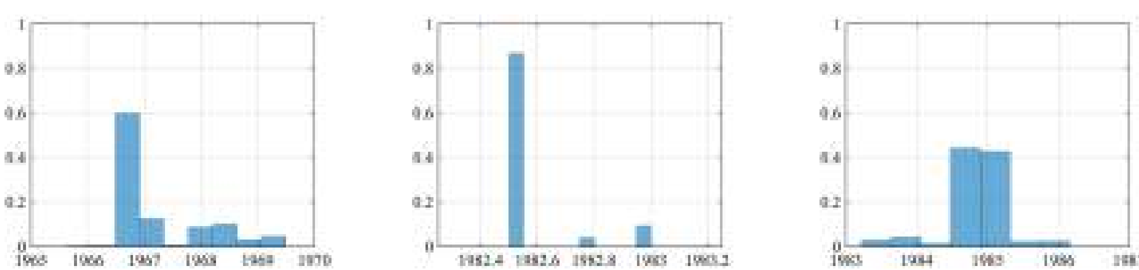


Figure A15: Posterior Distributions of Regime Switching Dates – Money Rule

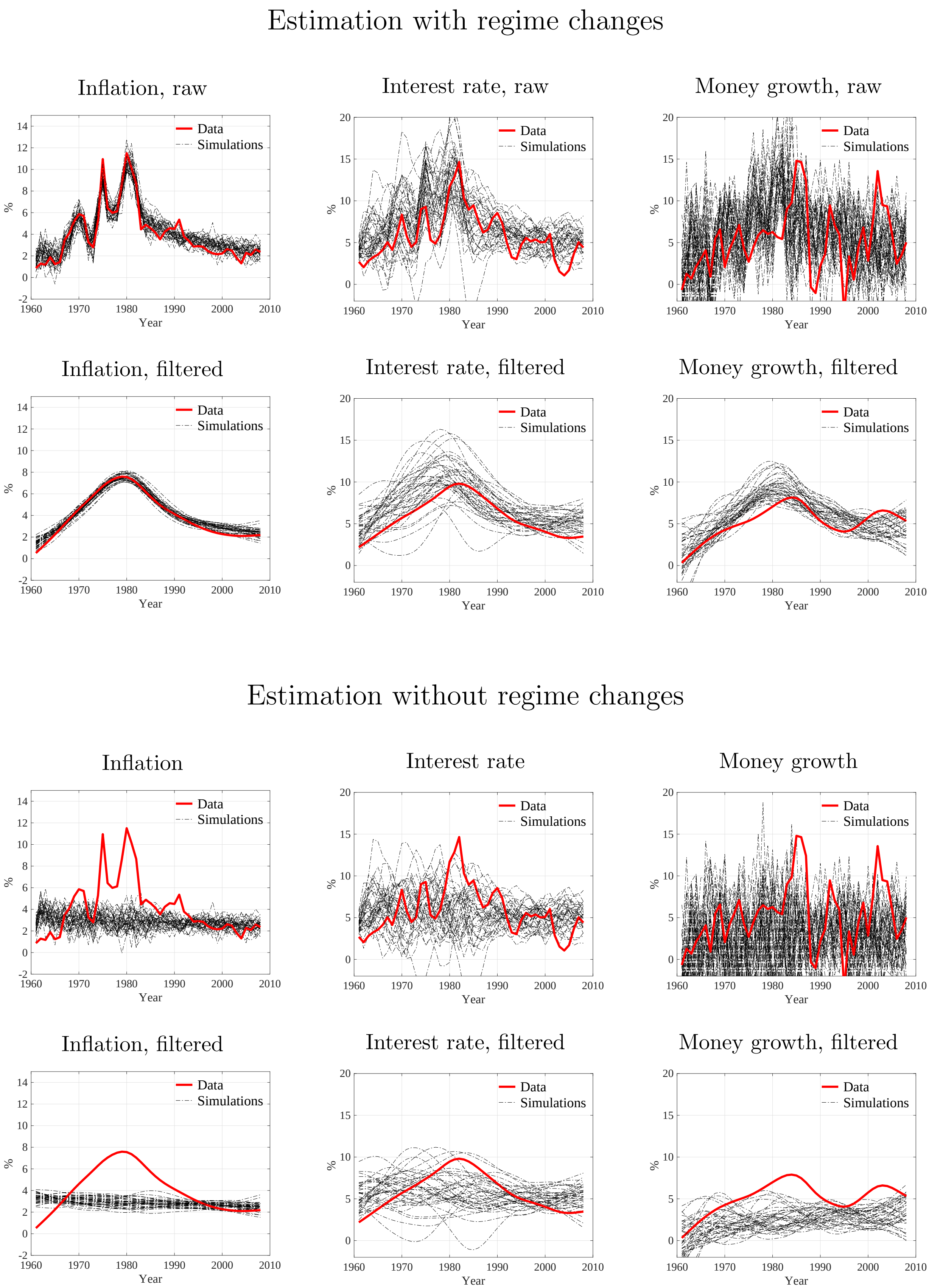
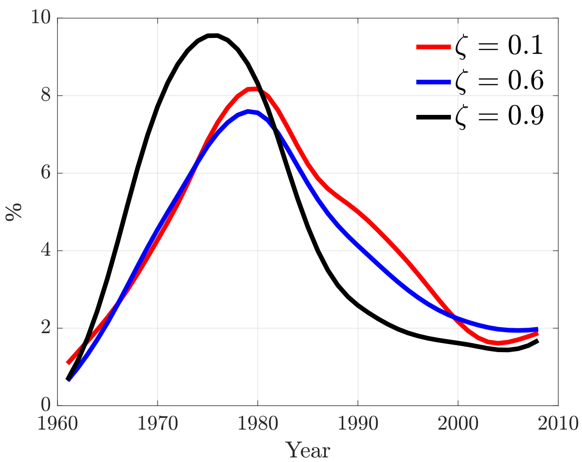
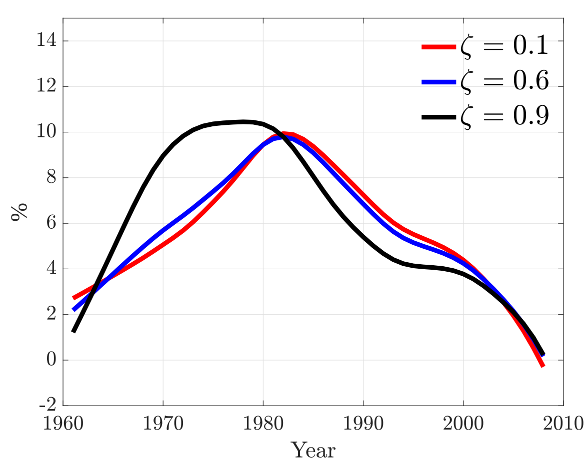


Figure A16: Model Fitness – Money Rule

(a) Simulation with regime changes

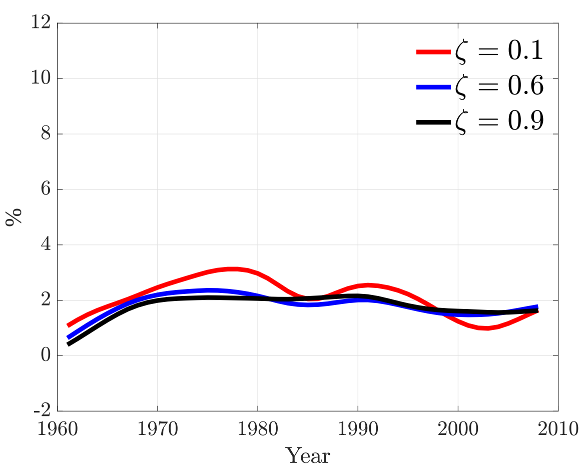


Inflation rates, HP filtered $\lambda = 100$

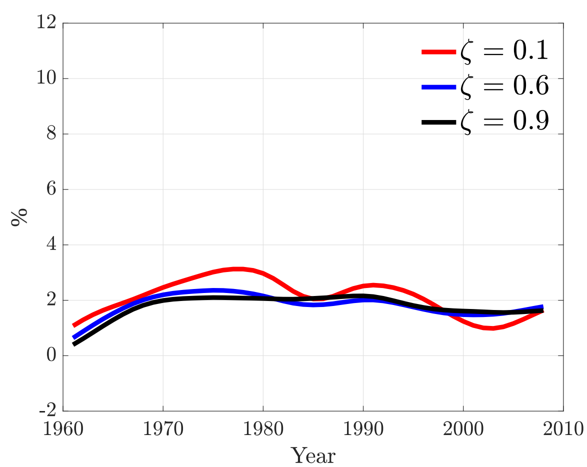


Nominal interest rates, HP filtered $\lambda = 100$

(b) Simulation without regime changes



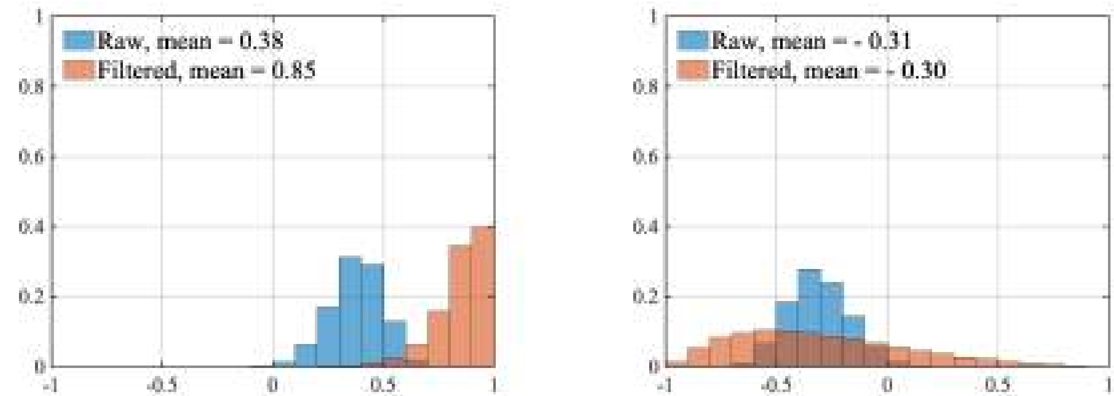
Inflation rates, HP filtered $\lambda = 100$



Nominal interest rates, HP filtered $\lambda = 100$

Figure A17: Sensitivity to Price Stickiness – Money Rule

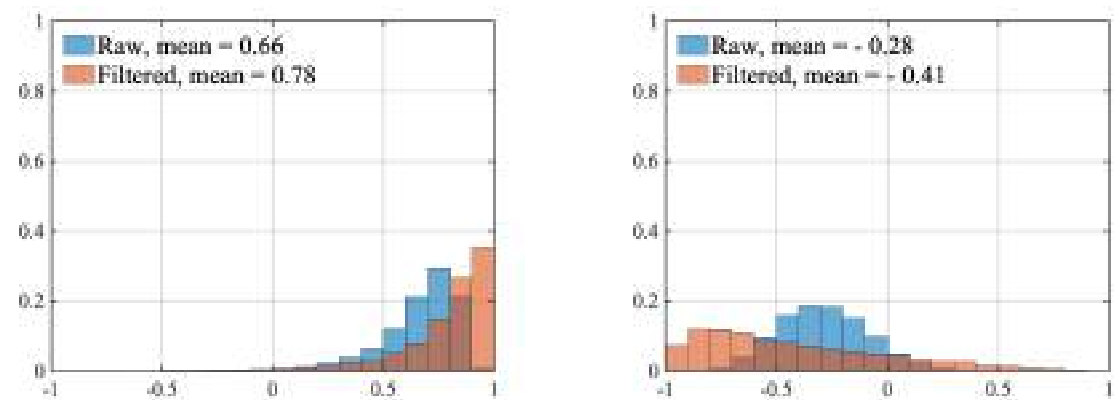
(a) Illustration 1



Benchmark with regime changes

Without regime changes

(b) Illustration 2



Benchmark with regime changes

Without regime changes

Figure A18: Correlations of Series in Simulated Data – Money Rule

D.2 Estimation with deterministic inflation target shock only

In our benchmark specification, we allow for stochastic inflation target shocks in regime 1. In this section, we estimate a model in which we only allow for a deterministic trend of inflation target.

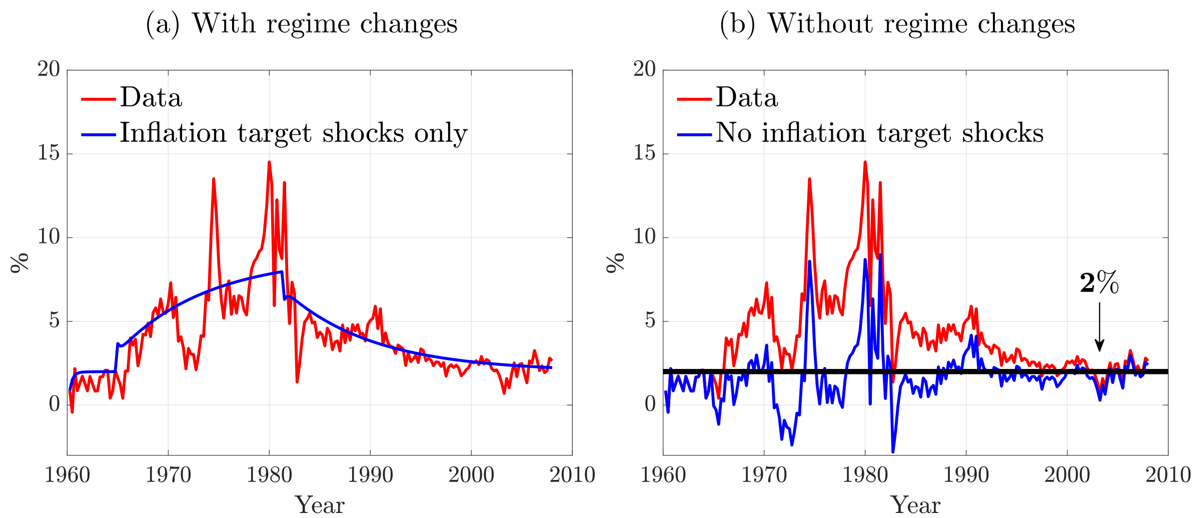


Figure A19: Estimation with Deterministic Inflation Target

Table A4: Variance Decomposition – Deterministic Inflation Target

	Regime 1: $\mathbb{I}^s = 0$					Regime 2: $\mathbb{I}^s = 1$				
	i_t	π_t	g_t	μ_t	x_t	i_t	π_t	g_t	μ_t	x_t
Shocks										
ε_i	3.4	7.1	6.8	8.3	26.1	3.4	7.1	6.8	6.2	26.1
ε_a	89.2	17.1	25.3	35.6	56.2	89.2	17.1	25.3	16.3	56.2
ε_e	7.4	75.8	1.1	1.1	17.8	7.4	75.8	1.1	1.0	17.8
ε_z	0.0	0.0	66.8	15.3	0.0	0.0	0.0	66.8	21.3	0.0
$\varepsilon_{\pi^\star}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ε_m	0.0	0.0	0.0	39.6	0.0	0.0	0.0	0.0	55.1	0.0

Table A5: Estimation Results: Deterministic Inflation Target

	Shape	Mean	Std Dev.	Mean	Mode	5%	95%
$100 \times \sigma_i$	Inv. Gamma	1	2	0.11	0.11	0.09	0.13
$100 \times \sigma_a$	Inv. Gamma	1	2	1.23	1.30	1.05	1.63
$100 \times \sigma_e$	Inv. Gamma	1	2	0.11	0.11	0.09	0.13
$100 \times \sigma_z$	Inv. Gamma	1	2	0.43	0.43	0.38	0.48
$100 \times \sigma_{\pi^\star}$	Inv. Gamma	1	2	0.09	0.09	0.08	0.10
$100 \times \sigma_\tau^{obs}$	Inv. Gamma	1	2	0.25	0.41	0.16	1.23
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.19	1.20	1.08	1.35
κ	Normal	2	0.3	2.28	2.30	2.10	2.49
κ_m	Normal	2	0.3	1.33	1.38	1.15	1.61
ρ_i	Beta	0.5	0.2	0.85	0.85	0.81	0.89
ϕ_π	Normal	2	0.5	2.42	2.45	1.82	3.09
ϕ_x	Normal	0.125	0.05	0.35	0.35	0.29	0.41
$10 \times \omega$	Normal	0.5	0.1	0.54	0.54	0.38	0.69
η	Normal	0.5	0.05	0.44	0.43	0.35	0.52
ρ_m	Beta	0.5	0.2	0.92	0.92	0.87	0.96
ρ_a	Beta	0.5	0.2	0.87	0.86	0.82	0.90
ρ_e	Beta	0.5	0.2	0.55	0.55	0.46	0.64
ρ_τ	Beta	0.5	0.2	0.83	0.84	0.76	0.91
ρ_π	Beta	0.5	0.2	0.97	0.97	0.97	0.98
Δ_π	Uniform	[-2 , 6]		1.72	1.72	1.47	1.98
ρ_ξ	Beta	0.5	0.2	0.70	0.71	0.57	0.83

(a) T^{on} (b) T^{off} (c) T_κ

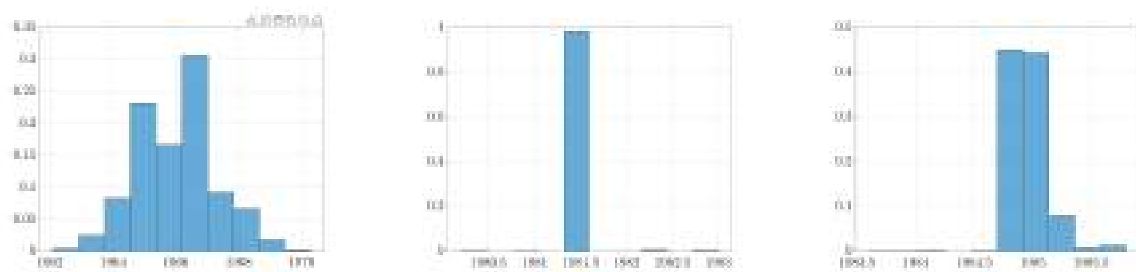


Figure A20: Posterior Distributions of Regime Switching Dates – Deterministic Inflation Target

D.3 Estimation with regime changes in monetary policy responsiveness

Recall the benchmark Taylor rule is given as

$$\begin{aligned} i_t =& i_t^* + \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t - y^*) + \varepsilon_t^i, \\ \pi_t^* =& (1-\rho_\pi)\pi^s(\mathbb{I}^s) + \rho_\pi \pi_{t-1}^* + \mathbb{I}^s \varepsilon_{\pi,t} \end{aligned}$$

with regimes

$$\mathbb{I}^s = \begin{cases} 0 & \text{for } t \in [1960, T^{\text{on}}) \quad \text{and} \quad t \in [T^{\text{off}}, \infty) \\ 1 & \text{for } t \in [T^{\text{on}}, T^{\text{off}}) \end{cases}$$

and π_t^* evolves in different regimes

$$\begin{aligned} \pi^s(0) &= 2\% \\ \pi^s(1) &= 2\% + \Delta \end{aligned}$$

We can alternatively estimate another model in which there is regime change in policy responsiveness parameters ϕ_π and ϕ_y, i.e.

$$i_t = i^* + \phi_\pi(\mathbb{I}^s)(\pi_t - \pi^*) + \phi_y(\mathbb{I}^s)(y_t - y^*) + \varepsilon_t^i,$$

where the evolution of responsive parameters follow:

$$\phi_\pi(0) = \phi_\pi, \quad \phi_\pi(1) = \phi_\pi^*, \quad \phi_y(0) = \phi_y, \quad \phi_y(1) = \phi_y^*$$

Table A6 and Figure A22 report estimates when we allow RS in responsiveness only and shut down RS in inflation target. Table A7 and Figure A23 report estimates when we allow RS in both.

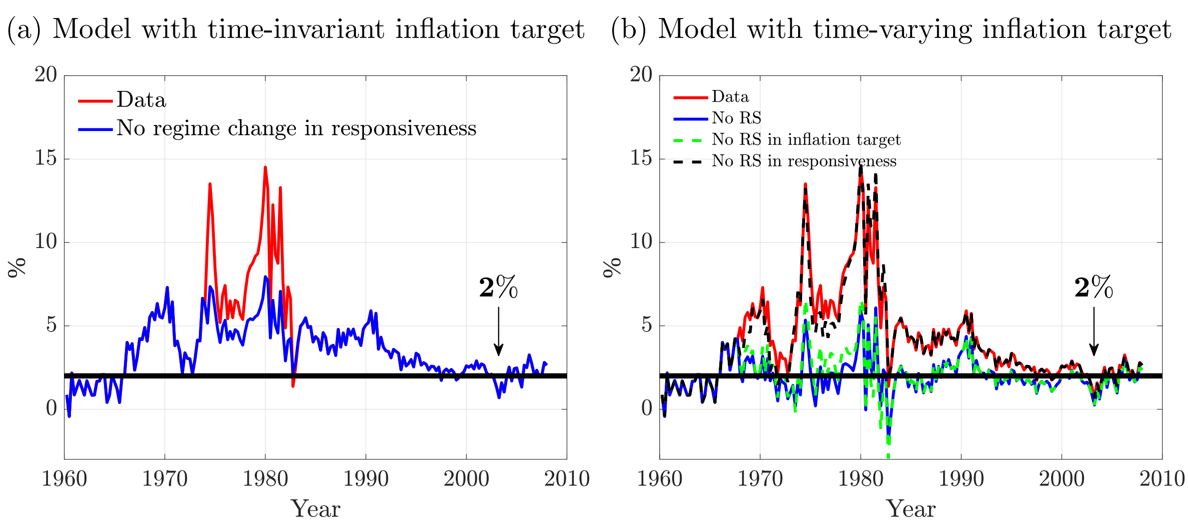


Figure A21: Role of Change in Responsiveness

Table A6: Estimation Results: Regime Switching in Responsiveness Only

	Shape	Mean	Std Dev.	Mean	Mode	5%	95%
$100 \times \sigma_i$	Inv. Gamma	1	2	0.14	0.14	0.12	0.16
$100 \times \sigma_a$	Inv. Gamma	1	2	1.30	1.31	1.04	1.73
$100 \times \sigma_e$	Inv. Gamma	1	2	0.08	0.08	0.06	0.09
$100 \times \sigma_z$	Inv. Gamma	1	2	0.46	0.45	0.40	0.52
$100 \times \sigma_\pi$	Inv. Gamma	1	2	0.12	0.11	0.10	0.13
$100 \times \sigma_\tau^{obs}$	Inv. Gamma	1	2	0.40	0.33	0.16	1.19
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.25	1.24	1.11	1.42
κ	Normal	2	0.3	2.14	2.13	1.95	2.34
κ_m	Normal	2	0.3	1.37	1.36	1.14	1.63
ρ_i	Beta	0.5	0.2	0.78	0.77	0.73	0.82
ϕ_π	Normal	2	0.5	3.72	3.67	3.22	4.24
ϕ_x	Normal	0.125	0.05	0.20	0.17	0.15	0.25
$10 \times \omega$	Normal	0.5	0.1	0.51	0.50	0.34	0.66
η	Normal	0.5	0.05	0.47	0.47	0.39	0.55
ρ_m	Beta	0.5	0.2	0.97	0.97	0.95	0.98
ρ_a	Beta	0.5	0.2	0.89	0.89	0.84	0.92
ρ_e	Beta	0.5	0.2	0.93	0.94	0.89	0.96
ρ_τ	Beta	0.5	0.2	0.70	0.72	0.59	0.81
ρ_π	Beta	0.5	0.2	0.51	0.50	0.19	0.82
Δ_π	Uniform	[-2 , 6]		2.05	1.50	-1.63	5.64
ρ_ξ	Beta	0.5	0.2	0.56	0.56	0.43	0.69
ρ_i^*	Beta	0.5	0.2	0.73	0.76	0.64	0.81
ϕ_π^*	Normal	2	0.5	1.61	1.88	1.28	2.05
ϕ_x^*	Normal	0.125	0.05	0.15	0.15	0.09	0.21

(a) T^{on} (b) T^{off} (c) T_κ

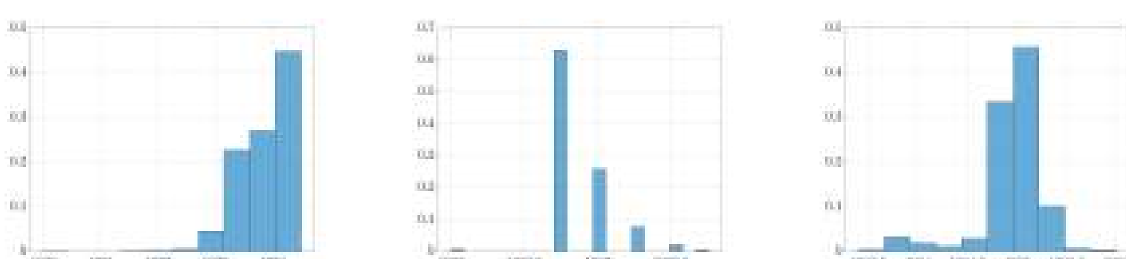


Figure A22: Posterior Distributions of Regime Switching Dates: Regime Change in Responsiveness Only

Table A7: Estimation Results: Regime Switching in Both Inflation Target and Responsiveness

	Shape	Mean	Std Dev.	Mean	Mode	5%	95%
$100 \times \sigma_i$	Inv. Gamma	1	2	0.08	0.08	0.06	0.10
$100 \times \sigma_a$	Inv. Gamma	1	2	1.35	1.40	1.11	1.67
$100 \times \sigma_e$	Inv. Gamma	1	2	0.11	0.11	0.09	0.14
$100 \times \sigma_z$	Inv. Gamma	1	2	0.48	0.48	0.43	0.54
$100 \times \sigma_\pi$	Inv. Gamma	1	2	0.10	0.10	0.09	0.12
$100 \times \sigma_\tau^{obs}$	Inv. Gamma	1	2	0.17	0.17	0.13	0.22
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.26	1.24	1.12	1.42
κ	Normal	2	0.3	1.94	1.90	1.75	2.13
κ_m	Normal	2	0.3	1.36	1.35	1.15	1.61
ρ_i	Beta	0.5	0.2	0.88	0.89	0.84	0.91
ϕ_π	Normal	2	0.5	2.53	2.40	1.81	3.25
ϕ_x	Normal	0.125	0.05	0.32	0.33	0.26	0.38
$10 \times \omega$	Normal	0.5	0.1	0.52	0.51	0.36	0.67
η	Normal	0.5	0.05	0.46	0.47	0.37	0.55
ρ_m	Beta	0.5	0.2	0.94	0.95	0.89	0.98
ρ_a	Beta	0.5	0.2	0.87	0.87	0.83	0.90
ρ_e	Beta	0.5	0.2	0.51	0.48	0.38	0.63
ρ_τ	Beta	0.5	0.2	0.78	0.80	0.67	0.88
ρ_π	Beta	0.5	0.2	0.98	0.98	0.97	0.98
Δ_π	Uniform	[-2 , 6]		1.14	1.14	0.22	2.06
ρ_ξ	Beta	0.5	0.2	0.61	0.61	0.46	0.78
ρ_i^*	Beta	0.5	0.2	0.92	0.95	0.86	0.97
ϕ_π^*	Normal	2	0.5	1.87	1.81	1.18	2.67
ϕ_x^*	Normal	0.125	0.05	0.24	0.23	0.17	0.30

(a) T^{on} (b) T^{off} (c) T_κ

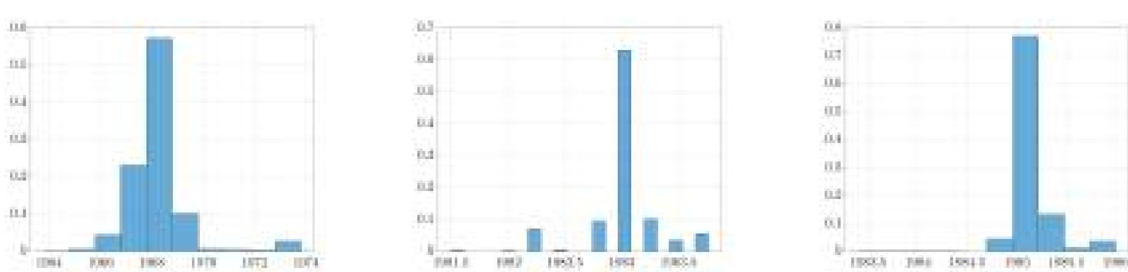


Figure A23: Posterior Distributions of Regime Switching Dates: Regime Switching in Both Inflation Target and Responsiveness

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